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ABSTRACT

This concept module on vibrations and waves is one of thirteen modules that provide a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, postsecondary technical schools. Each of the thirteen concept modules discusses a single physics concept and how it is applied to each energy system. This module examines wave motion in general and discusses common examples of vibrations and waves. Included with the concept module are seventeen application modules that discuss a specific application of a single concept in a single energy system, and contain a student laboratory exercise. Laboratory modules included are Speed of Sound in Materials, Reflection and Absorption of Sound Energy, Vibration Isolation, Resonance, Sound Speakers, Cavity Vibrations and Standing Waves, Speaker Enclosures, Reverberation Time, Doppler Effect, Pressure Measurement of Sound Waves, Alternating Voltage and Current, Phase Relationships in Alternating Circuits, Oscilloscope, Rectifiers, Electronic Filtering in Direct Current Power Supplies, Electronic Filtering for Frequency Selection, and Modulation of Electrical Waves. (LRA)

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UTC

UNIFIED TECHNICAL CONCEPTS

CONCEPT MODULE 11-0

ED190759

VIBRATIONS AND WAVES

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
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INTRODUCTION TO UTC

Complex, modern equipment often contains combinations of hydraulic, pneumatic, thermodynamic or optical subsystems, as well as mechanical and electrical components. These subsystems and components appear not in isolation but as interdependent parts of complex systems. Consequently, a modern technician must understand not only the separate subsystems but also their interrelationships and the roles they play in an integrated system.

Since physics provides the foundation for modern technology, physics instruction in technical programs cannot be overemphasized. Physics courses for technicians should achieve three major objectives:

- Provide students with all the basic physics principles that apply to their chosen field
- Reinforce these principles through the use of "real" examples and hands-on experience.
- Organize knowledge and skills gained in a manner that provides ease of application in solving systems-oriented problems.

These objectives must be accomplished in a minimum of instructional time, and in a manner flexible enough to meet the needs of a variety of technical specialties.

Unified Technical Concepts (UTC) is a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, post-secondary technical schools. UTC organizes technical physics into 13 basic concepts (Force, Work, etc.) that apply in four "energy systems" (Mechanical, Fluidal, Electrical, Thermal). Each of the 13 "concept" modules discusses a single physics concept and how it is applied to each energy system. Similarly, each "application" module discusses a specific application of a single concept in a single energy system, and contains a student laboratory exercise.

The recommended prerequisite for UTC is one year of high school algebra, but eight supplementary modules are available for students requiring additional skills or review in the areas of math, units and graphing.

UTC is an alternate method of presenting what heretofore has been a formidable and sometimes confusing subject for technicians. It provides laboratory experiments that apply to specific technological fields, which allows a student to relate more quickly to module content - to understand how it applies to a specific technology.

UTC also represents a unique approach to teaching applied physics. Its flexible modular format allows it to be tailored by physics and technology instructors to meet

specific needs of students in a specific technology.

STRUCTURE OF UTC

The modern world is based on the control and application of energy. Technology is the means by which that energy is applied to accomplish a desired task. Technical physics, being primarily a study of the physical basis of the control and use of energy, can be categorized according to the type of energy involved. Thus, in UTC, physics is divided first into four "energy systems" - Mechanical, Fluidal, Electrical and Thermal.

The same divisions are characteristic of traditional approaches to teaching technical physics. A student usually begins by studying mechanics. Once this branch of physics (energy system) is mastered, electromagnetism, thermodynamics and hydraulics are studied (one energy system at a time).

UTC takes a different approach.

CONCEPTS IN PHYSICS

The basic ideas, or "concepts," of physics appear in all energy systems, and these concepts relate to one another in similar ways in all systems. UTC unifies technical physics by presenting first the physics concept and then its application in each energy system.

The first concept studied, for example, is that of force, - the physical quantity that produces motion. The basic characteristics of force are followed by a discussion of how this concept applies to "forcelike quantities" in the following energy "systems":

- In mechanical systems <u>force</u> and <u>torque</u> cause motion of mass.
- In fluidal systems <u>pressure difference</u> causes motion of fluids.
- In electrical systems <u>potential difference</u> causes motion of mass.
- In thermal systems <u>temperature difference</u> causes motion of heat energy.

Problems, demonstrations and hands-on laboratory exercises are used to show the application of the concept in each energy system, and to specific devices or processes common in modern technology.

Thus, UTC provides a student with the basic concepts of physics reinforced by practical application and organized into a consistent whole; UTC is readily applied to future technical problems, whether in other courses or on the job.

COMPONENTS OF UTC

UTC incorporates three types of modules.

CONCEPT MODULES

Thirteen concept modules form the backbone of UTC (each presents a single concept in applied physics and discusses the application of that concept in each energy system):

1-0 Force.
2-0 Work.
3-0 Rate.
4-0 Momentum.
5-0 Resistance.
6-0 Power.
7-0 Potential and Kinetic Energy.
8-0 Force Transformers.
9-0 Energy Convertors.
10-0 Transducers.
11-0 Vibrations and Waves.
12-0 Time Constants.
13-0 Radiation.

APPLICATION MODULES

Each of the 170 application modules discusses the application of a single physics concept in a single energy system, and each module includes a laboratory exercise in which students investigate the application. For example, 5E5 "Resistance of Wires" (located under Concept 5-0 "Resistance"), discusses the electrical resistance of wires. In the laboratory, students calculate and measure experimentally the resistance of various wires.

SUPPLEMENTARY MODULES

In UTC, mathematics has been reduced to a minimum: one year of high school algebra is adequate preparation. The following eight supplementary modules are available for students requiring additional preparation or review:

S1 Formula Interpretation.
S2 International System of Units.
S3 Logarithms and Exponents.
S4 Angles and Triangles.
S5 Vectors and Scalars.
S6 Dimensional Analysis.
S7 Reading and Drawing Graphs.
S8 Precision, Accuracy, and Measurement.

STRUCTURE OF A UTC MODULE

Each module in UTC contains seven basic elements:

INTRODUCTION

A brief description of the topics to be discussed, and their importance in technology, introduces each module.

PREREQUISITES

A list of prerequisite UTC modules, and notice of any special knowledge or skills required for successful completion of the module, are given.

OBJECTIVES

Specific tasks that a student should be able to accomplish after completing the module are stated in behavioral terms; objectives also are the basis of testing for UTC.

DISCUSSION

This segment is an explanation of the topic to be studied. It contains the appropriate physics as well as examples of its application. The discussion includes development of equations, example problems, figures and tables of useful data. It addresses each of the objectives (except those pertaining directly to a laboratory experiment).

LABORATORY (called DEMONSTRATIONS in the Concept Modules)

The laboratory section of UTC modules consists of the following three parts:

- "Materials." A list of all equipment and supplies required for the laboratory.
- "Procedure." A step-by-step set of instructions for performing the experiment.
- "Data." An organized format for recording all necessary data and observations.

SELF CHECK

A set of questions and problems based on the module objectives is included. It is intended for use by students to check their own understanding of module content.

REFERENCES

This list of sources directs students to additional information on topics discussed in the module.

TABLE OF CONTENTS

11-0	Vibrations and Waves
11M1	Speed of Sound in Materials
11M2	Reflection and Absorption of Sound Energy
11M3	Vibration Isolation
11M4	Resonance
11M5	Sound Speakers
11F1	Cavity Vibrations and Standing Waves
11F2	Speaker Enclosures
11F3	Reverberation Time
11F4	Doppler Effect
11F5	Pressure Measurement of Sound Waves
11E1	Alternating Voltage and Current
11E2	Phase Relationships in AC Circuits
11E3	Oscilloscopes
11E4	Rectifiers
11E5	Electronic Filtering in DC Power Supplies
11E6	Electronic Filtering for Frequency Selection
11E7	Modulation of Electrical Waves

INTRODUCTION

Energy can be transferred from one place to another in a variety of ways. A bowler, for example, transfers kinetic energy of the rolling ball to the bowling pins. Here, one object, the ball, interacts with another, the pin. In other instances, energy moves in the form of waves, without the transfer of bulk matter. Sound, for instance, is a wave that carries energy from a source (megaphone) to a receiver (ear). Light is a wave that carries energy from a source, such as the sun, to a receiver, such as the eye. Wave motion always is characterized by a vibration or displacement at some location (source) and is propagated through a medium (as sound through air) or through a vacuum (as light through space). In some wave motion, the vibration is along the direction of propagation and is described as "longitudinal wave motion." In other instances, the vibration is perpendicular to the direction of propagation and is described as a "transverse wave motion." Sound energy propagates as a longitudinal wave, and light energy propagates as a transverse wave. This module examines wave motion in general and discusses common examples of vibrations and waves.

PREREQUISITES

Student should have completed Concept Module 6-0, "Power."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Distinguish between longitudinal and transverse waves by giving at least two examples of each type and by drawing and labeling a sketch of each.

- Define the following words associated with waves and wave motion:

  Propagating medium.
  Wavelength.
  Frequency.
  Period.
  Displacement.

Amplitude.
Phase.
Standing wave.
Constructive interference.
Destructive interference.
Beats.

- Calculate the wavelength of a wave, given its velocity and frequency.

- Interpret this equation, explaining the significance of each symbol:

$$y = A \sin 2\pi \left(\frac{x}{\lambda} - ft\right).$$

- Explain the meaning of the expression "the current leads the voltage by a given phase angle" by using sine-wave sketches of both current and voltage.

- Describe the superposition principle.

- Describe wave phenomena in each of the following energy systems:

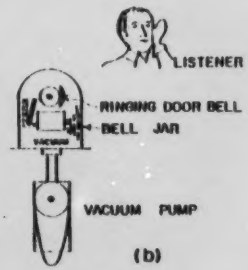
  Mechanical.
  Fluidal.
  Electrical.

DISCUSSION

A wave is a disturbance in which energy is transferred from one location to another. Some well-known examples of waves are sound waves that result from vibrating guitar strings, seismic waves that move through the earth following an earthquake, water waves produced by the vertical motions of the surface of a lake, and light waves that can travel through a vacuum.

Several types of waves require a material in which to travel, or propagate, themselves. This material is referred to as a "medium." Sound is an example of a wave that requires a medium through which to move. The propagation of sound waves through air is a fact of common experience. Sound also can travel through liquids and solids. Sound cannot travel through a vacuum (Figure 1).

(a)



(b)

FIG. 1

Electromagnetic waves do not require a medium. Light, for example, is an electromagnetic wave that reaches the earth from the sun through the vacuum of outer space. Other waves of this type are radio waves, X-rays, and heat waves.

The energy transmitted by a wave may be in the form of a single burst, or pulse, or it may be in the form of many identical pulses, evenly spaced. Consider a person holding one end of a rope fastened to a wall on the other end (Figure 2). The person makes one up-and-down motion with an arm. This motion is transmitted to the left end of the rope, which then transmits the motion gradually down the entire rope length. The result is the generation of a single pulse wave, which moves down the rope (Figure 2).

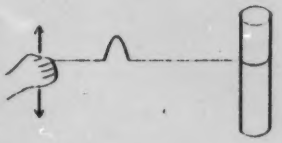


FIG. 2

When the person moves an arm up and down repeatedly, the wave produced is made up of a series of identical pulses (Figure 3). When wave pulses are repeated at regular intervals, the wave is called a "periodic wave."

A shock wave moving away from the scene of an explosion is an example of sound energy being transferred in the form of a single pulse through the air. A musical tone coming from a vibrating guitar string is an example of a continuous wave of sound energy.

FIG. 3

TRANSVERSE WAVES

"Transverse" waves are those in which the disturbance producing the wave moves perpendicularly to the direction in which the disturbance is propagated. The wave motion produced in a rope (Figure 3) is one example.

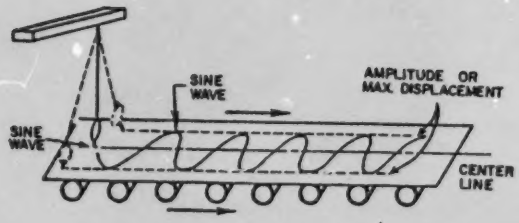


FIG. 4

Here, a sine wave will be used as an example of transverse wave motion. Figure 4 shows a sine wave being produced by a swinging pendulum to which a pencil is fastened. A piece of paper is pulled under the pendulum at a constant rate of speed, while the pencil traces the path of the swinging pendulum. Motion that generates a sine wave (Figure 4) is called "simple harmonic motion."

Displacement at any time is the distance of the pendulum from its center, or equilibrium, position (Figure 4). The maximum displacement of the pendulum is called the "amplitude" of the wave. The energy transferred by the wave turns out to be proportional to the square of the amplitude. The wavelength λ is the distance between identical points of the wave — from crest to crest, for example. Both amplitude and wavelength are illustrated in Figure 5.

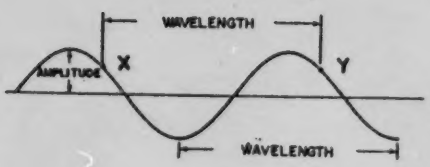


FIG. 5

The pulses that make up periodic waves can have many shapes. Several pulse shapes are illustrated in Figure 6.

The frequency is the number of complete waves produced each second. When one wave is produced each second, the wave has a frequency of one hertz (Hz).

The time required for one complete wave pulse is known as the "period" of the wave. The period and the frequency are reciprocals of each other, as indicated by Equation 1.

$$T = \frac{1}{f}$$

or

$$f = \frac{1}{T} \qquad (1)$$

where

T = period of wave.
f = frequency of wave.

The position and motion of a given point on a wave defines the phase of the wave. Points that have the same displacement and that are moving in the same direction are in phase with each other. Thus, points x and y on the wave (Figure 5) are in phase.

LONGITUDINAL WAVES

A "longitudinal wave" is one in which the disturbance producing the pulses moves parallel to, or along the direction in which, the disturbance is propagated. If a slightly-stretched spring were attached to a barely-swinging pendulum in such a way that the pendulum would swing parallel to the spring (Figure 7), a longitudinal wave would be produced.

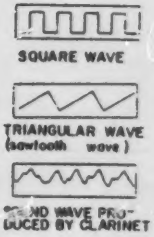


FIG. 6

As illustrated in Figure 7, when the pendulum has moved to its maximum right-hand position, a compression of the springs is produced. Similarly, when it has moved to the left-hand position, an elongation, or "rarefaction," is produced. This series of compressions and rarefactions produced represents a periodic longitudinal wave moving from left to right in the spring. Sound waves are of this type.

A comparison can be drawn between the compression and rarefactions of a longitudinal wave and the crests and troughs of a transverse wave (Figure 8).

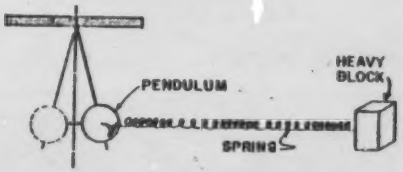


FIG 7

The definitions for amplitude, wavelength, frequency, and period presented earlier for transverse waves apply equally well to longitudinal waves. For example, the wavelength λ is the distance between adjacent compressions.

WAVE VELOCITY

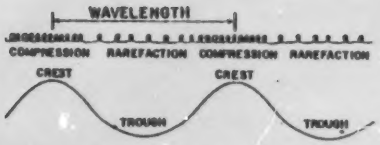


FIG. 8

A wave pulse, or disturbance, moves a distance equal to its wavelength in a time equal to the period of the wave. Since velocity is distance divided by time, the relationship expressed by Equation 2 holds true for all wave motion.

$$v = \frac{\lambda}{T} \qquad (2)$$

where

v = wave velocity.
λ = wavelength.

But since $T = \frac{1}{f}$, the value obtained is $v = \frac{\lambda}{1/f}$, or —

$$v = f\lambda \qquad (3)$$

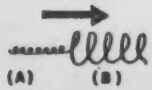


FIG. 9

The speed of a wave is affected greatly by the medium thorugh which the wave travels. For example, the sound of a moving train travels faster through the rails of the track than through air. Even in air, the speed of sound depends upon temperature and pressure of air. Other characteristics of a medium that can affect the speed of sound or light waves through that medium include density, elasticity, and index of refraction.

Waves traveling through a uniform medium do not change velocity, but waves that travel from one medium to another medium do experience velocity changes. This can be illustrated by the longitudinal wave motion in springs. Consider a wave moving in a moderately-stiff spring (Figure 9). When the wave pulse from the moderately-stiff spring reaches the boundary between the two springs, some of the energy in the pulse will be transferred into the very stiff spring, where it will continue to travel at a slightly different velocity. The remainder of energy will be reflected by the boundary and will bounce back in the moderately-stiff spring. Thus, some of the energy is reflected, and some is transmitted. The same effect occurs, but in different proportions, if the wave travels from the stiff spring into a very soft spring.

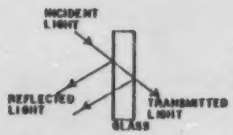


FIG. 10

A similar effect can be seen when a beam of light waves is directed on a flat piece of glass (Figure 10).

When a light wave strikes a flat piece of glass, some of the light enters the glass, and some is reflected. Again, some of the light passing through glass is reflected when light strikes the rear surface of the glass, and some exits the glass and passes into the air. The amounts of light reflected and transmitted will depend, among other things, upon the angle of incidence. Notice that reflection and change of velocity occur when the medium changes, first from air-to-glass, and then from glass-to-air.

FIG. 11

THE WAVE EQUATION

Consider the motion of a single point of the medium as a wave passes that point. The motion of a float on a fishing line because of a water wave (Figure 11) is a good example. The float, moving more-or-less vertically up and down, is shown riding the valley and, later (dotted line), riding the crest of the wave. As the wave disturbance moves past the float, it undergoes a vertical displacement shown by the arrow (Figure 12). Notice that the motion of the float is perpendicular (up-and-down) to the direction in which the wave moves (left-to-right). The wave shown is necessarily, then, a transverse wave.

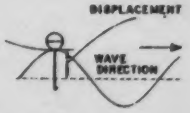
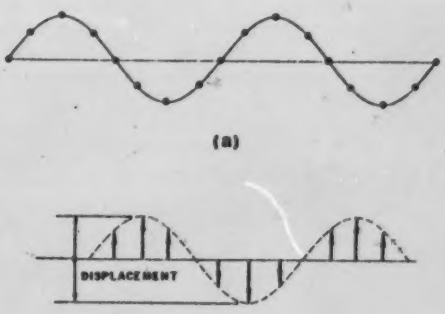


FIG. 12

The displacement of each point along a wave can be analyzed in the same manner. If a picture is taken of a moving wave, the photograph will show that each point along the wave has a different displacement. At the instant the photograph is taken, some of the points will be displaced (Figure 13a). However, each point will be seen to oscillate vertically with a periodic motion as time passes.

The momentary displacement of each point on the wave of Figure 13a is indicated by the arrows in Figure 13b. The direction of the arrow is the same as the direction of the displacement, and the length of the arrow indicates the amount of displacement of the point.

A mathematical analysis of wave motion leads to the general equation (Equation 4) for the displacement.



(a)

(b)

FIG. 13

$$y = A \sin 2\pi \left(\frac{x}{\lambda} - ft\right) \quad (4)$$

where

y = wave displacement.
A = wave amplitude.
f = frequency of wave.

Often, a sine wave can be understood readily when it is compared to wave motion around a circle. This relationship is pictured in Figure 14. Each angle drawn on the reference circle is 30°, or π/6 radians. Each division on the sine wave axis also is 30°, or π/6 radians. Notice how the points on the reference circle correspond to points on the wave. Thus, point 1 on the wave corresponds to 0 radian; point 2 corresponds to π/6 radian; point 3, to π/3 radian; and so on. One complete rotation on the reference circle corresponds to one complete sine wave. Now the phase of a point on the wave can be related to an angle. For example, point 6 on the wave has a phase of 5/6 π, and point 9 has a phase of 4/3 π.

Commonly 60-hertz alternating current (a.c.) electrical power produced in the United States is an example of wave motion applied to electricity. The voltage and current in an a.c. circuit varies with time according to a sine wave. The relationships for voltage and current as a function of time are given in Equations 5 and 6, respectively:

$$V_t = V_{max} \sin (2\pi ft) \quad (5)$$

where

V_t = instantaneous voltage.
V_{max} = maximum voltage.

$$I_t = I_{max} \sin (2\pi ft) \quad (6)$$

where

I_t = instantaneous current.
I_{max} = maximum current.

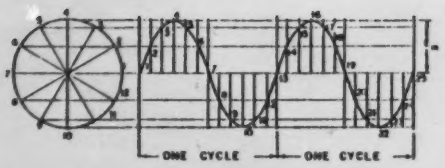


FIG. 14

Notice the similarities between Equations 4, 5, and 6. Many authors show Equations 5 and 6 as $V_t = V_{max} \sin \omega t$ and $I_t = I_{max} \sin \omega t$ (where $\omega = 2\pi f$ = "angular frequency"). Since 2π radians represents a full circle, 2πf is the angular velocity at which the reference circle in Figure 14 is repeated.

In a.c. circuits that contain only pure resistance, the voltage and current values at any instant of time (t) are "in phase" (Figure 15). In other words, both voltage and current values peak and trough in step.

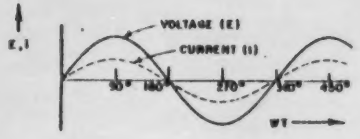


FIG. 15

When an a.c. circuit contains inductance and capacitance, the voltage and current may not be in phase. The angular difference between voltage and current is called the "phase angle." An out-of-phase condition is displayed in Figure 16. The phase angle θ shown in Figure 16 is 45°, or π/4 radians. The current is leading the voltage by 45° in this example. The voltage can lead the current (Figure 17). The phase angle θ can vary from 0° to 360°.

SUPERPOSITION

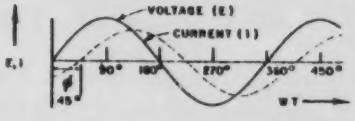


FIG. 16

When two or more waves traveling in the same medium overlap, a new wave is produced — a combination of the original waves. The "superposition principle" states that "the resultant wave will be the sum of the two component waves." Figure 18 illustrates the superposition of two waves. (The exact composite wave illustrated here only exists for the instant the component waves have the form shown.)

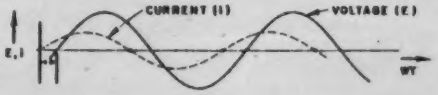


FIG. 17

At position 1, displacement n belongs to wave 1, and displacement m belongs to wave 2. The two displacements are added to give the displacement m + n of the composite wave. At position 2, a displacement of p is produced by both waves, so that the composite displacement at position 2 is equal to p + p, as shown. In position 3, no displacement is produced by wave 2. The composite displacement, therefore, is equal to the displacement of wave 1. Position 4 shows that wave 2 has a negative displacement,

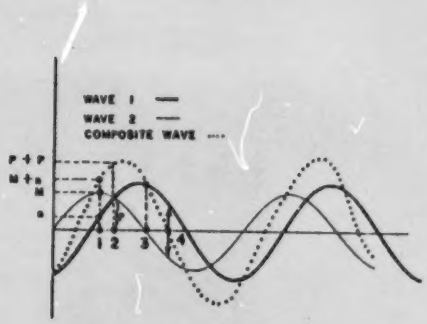


FIG. 18

and that wave 1 has an equal positive displacement. Here the two displacements counteract each other to give zero net displacement. A similar analysis can be conducted at any number of desired positions to verify that the composite wave (Figure 18) is the result of the superposition of waves 1 and 2.

STANDING WAVES

An important application of the superposition principle occurs in the formation of standing waves. A standing wave is produced when a train of reflected waves are superpositioned with an identical train of incident waves. The resultant wave will appear not to be traveling, and thus is called a "standing wave."

A standing wave can be illustrated by a single pulse formed in a rope firmly attached at one end (Figure 19). A pulse is inverted if it reflects from the fixed end. When an incident pulse and an identical reflected pulse pass each other, their displacements cancel; therefore, zero displacement of the rope results during the time the pulses are in contact. After the pulses cross, they continue with their initial shape and direction.

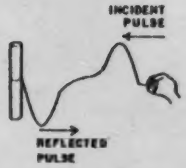


FIG. 19

When there are two continuous wave trains of the same wavelength and amplitude, but traveling in opposite directions, the composite wave appears to stand still. In this case, the two component waves do not add to zero everywhere at all times. Figure 20 shows the composite wave at four different instants.

The composite wave pattern shows points on the axis of the wave at which no displacement ever is observed. These points are called "nodes" and are one-half wavelength apart. The "antinodes" are points, also $\lambda/2$ apart, on the axis that correspond to maximum displacements of the standing wave.

Standing waves are produced in musical instruments, such as guitars or violins, where the disturbance generated by the musician travels backwards and forwards between the fixed ends of the wire. A pipe organ produces similar standing waves in pipes. The length and diameter of the pipes determine the characteristics

of the standing waves and, thus, the characteristics of the music produced.

INTERFERENCE

Interference is the resultant cancellation or reinforcement that occurs when two or more waves are superposed. If two identical waves meet in phase (crest meeting crest and trough meeting trough), the principle of superposition predicts that a wave of twice the amplitude will result. Similarly, if the two waves meet out of phase by 180° ($\lambda/2$), then a crest of one wave meets a trough of the other wave, and zero amplitude results. Figure 21 illustrates two identical wave sources oscillating in step in such a manner as to project waves of equal wavelength and amplitude. The solid lines represent crests of the wave; the dotted lines correspond to troughs.

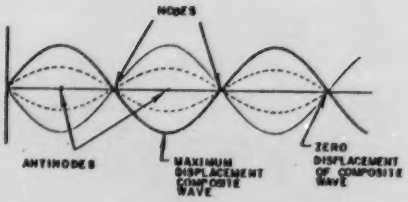


FIG. 20

At point 1, two troughs meet and produce a double amplitude. Point 2 is a position at which two crests meet and produce a double amplitude. Similarly, waves from the two sources meet in phase at points 3, 4, 5, 6, 7, and 8. These points are said to be points of "constructive interference." Thus, constructive interference occurs along the directions of OP, OP_2, and OP_2'. "Destructive interference" occurs at point 9, 10, 11, 12, 13, 14, 15, and 16, because a wave crest from one source meets a wave trough from the other source.

Since a crest and trough are 180° out of phase, zero displacement results at these positions. Destructive interference occurs along the directions OP_1 and OP_1'.

Constructive and destructive interference is observed when two sources generate waves of equal wavelength or freuqency. If the sources produce waves of slightly different frequencies, a phenomenon known as "beats" will occur. Consider two slightly

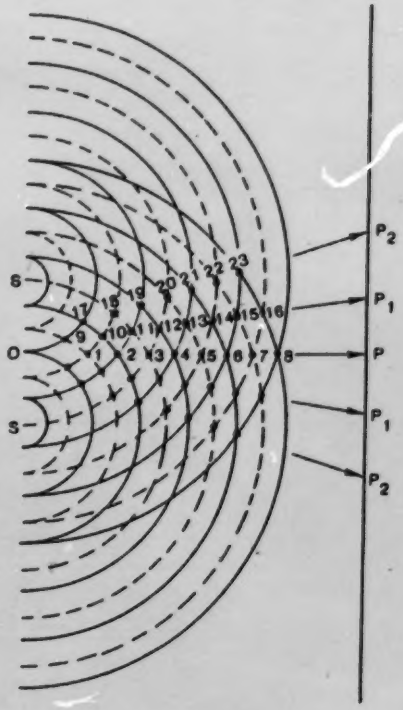


FIG. 21

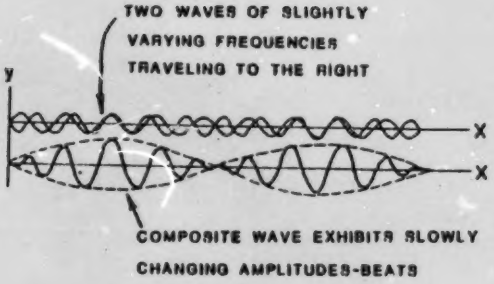


FIG. 22

different frequency waves traveling in the same direction (Figure 22). Addition of the two component waves of similar frequency is shown along the lower axis. Notice that where the component waves are in phase (or nearly so), the amplitude of the composite wave is large; and where the component waves are out of phase (or nearly so), the amplitude of the composite wave is small. The result shows two "pulses," or "beats." If the pulses are not too rapid, they can be detected separately by the human ear. Musicians listen for beats when tuning their instruments. The beat frequency turns out to be equal to the difference in frequency of the two component waves. Thus, if no beats occur, the instruments are in tune.

WAVE MOTION AS A UNIFYING CONCEPT

The basic characteristics of waves discussed in this module apply to all wave phenomena. The following are important examples of waves in several energy systems. The exception here is the thermal system, which exhibits no wave motion.

MECHANICAL SYSTEMS

In mechanical systems, both longitudinal and transverse waves are important. Longitudinal mechanical waves are compression waves in a material: Sound waves travel through a solid; compression waves are produced by a moving spring; and mechanical vibrations are present in machines and in structures. Transverse mechanical waves include the side-to-side motion that occurs in the strings of musical instruments and in torsional (twisting motion) waves. In mechanical structures, the reduction or elimination of wave motion often is a primary design consideration.

FLUIDAL SYSTEMS

The most important fluidal waves are compression waves (sound) in gases. The field of acoustics is based upon the study and control of these waves. The design of sound-delivery systems, for example, requires an analysis of the patterns of the radiated sound energy, the interference of sound waves from two or more sources, and the absorption and reflection of sound waves by materials.

ELECTRICAL WAVES

Electrical waves are among the most important to modern technology. Commercial electrical power is delivered as an electrical wave at a frequency of 60 Hz. Most electronic equipment performs its function, at least in part, through the manipulation of electrical waves. The concepts of superposition of waves and phase difference are particularly important in the analysis of electronic circuits.

X-rays, light, microwaves, and radio waves are examples of electromagnetic radiation. This type of transverse wave consists of oscillating electric and magnetic fields traveling through space; it is discussed in detail in Concept Module 13-0, "Radiation."

SUMMARY

A wave is a periodic disturbance in either a medium, such as air, or in a vacuum, such as space. Some waves require a medium; for example, sound waves require air through which to travel. Light waves and other electromagnetic waves do not require a medium. Wave energy can be transmitted in the form of a single pulse or in many evenly-spaced pulses.

Transverse waves are produced when the source moves perpendicularly (from side to side) to the direction in which the waves travel. Longitudinal waves are propagated when the individual parts of the source vibrate back and forth in the direction in which the waves travel.

Wave velocity is affected by the material through which waves travel. Two or more waves are superimposed when they overlap within the same medium to produce standing waves or interference.

Wave motion applies, as a unifying concept, to mechanical, fluidal, and electrical systems.

WAVES WITH OR WITHOUT A MEDIUM

MATERIALS

Bell jar.
Vacuum pump.
Door bell.
Dry cell.
Sound level meter (optional).
Flashlight or small laser.
Light meter (optional).

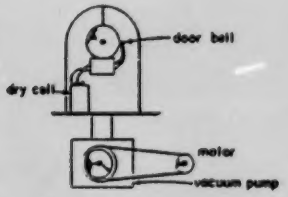


FIG. 23

PROCEDURE

1. Arrange door bell buzzer to operate under bell jar in place on vacuum pump. Figure 23 illustrates this experimental arrangement. Suspend buzzer inside bell jar or cushion it in such a manner that sound cannot be carried by conduction through solid material. Observe sound level of buzzer before air is removed from bell jar, and compare this level to sound level after air is removed. If sound level meter is available, you may use it to obtain more accurate comparisons. How well does sound travel through a vacuum?

2. Shine beam of light from flashlight through glass bell jar. Use light meter to compare amount of light transmitted through air with amount transmitted through vacuum. Does light require a medium for transmission?

WAVES IN A SLINKY

MATERIALS

Slinky.
Meter stick.
Polaroid camera (optional).
Stopwatch.

PROCEDURE

1. Place slinky on lab table. Hold one end fixed while generating single transverse pulse at other end. Sketch shape of generated pulse and of reflected pulse.

2. Produce both transverse and longitudinal pulses in slinky and describe difference.

3. Generate transverse periodic wave in slinky, and determine its wavelength: place meter stick near slinky, and take a picture of wave with polaroid camera. Use stopwatch to determine period (T) of wave. Calculate velocity of wave, using Equation 2.

4. Adjust frequency of periodic waves until standing-wave pattern is observed. Draw sketch of standing wave pattern.

SUPERPOSITIONING OF SOUND WAVES

MATERIALS

Two identical loudspeakers.
Two sine-wave generators to drive the speakers.
Microphone.
Oscilloscope or portable tape recorder.
Laboratory stands for mounting speakers.
Connecting wires.

PROCEDURE

1. Operate two identical loudspeakers connected in same way to a sine-wave generator. A small amplifier may be necessary. Connect microphone to oscilloscope in order that sound generated by loudspeakers can be viewed on oscilloscope screen. Position loudspeakers facing each other (Figure 24). Adjust distance between speakers and frequency until a measurable standing-wave pattern is produced between them. Detect standing-wave pattern by observing changes in amplitude (nodes and antinodes) of wave on oscilloscope as microphone is moved from one speaker to the other on a line between them. Have your instructor verify your results.

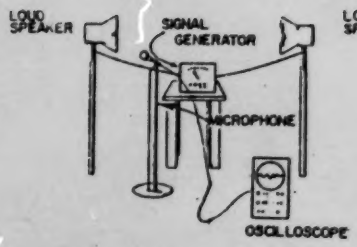


FIG. 24

2. Rotate loudspeakers 90° to face same direction. Move microphone throughout space in front of speakers, and identify areas of constructive interference and areas of destructive interference. Have your instructor verify your results.

3. Operate each speaker from a separate signal generator. Vary the frequency of one signal generator until you hear beats. The oscilloscope and microphone are recommended, but are unnecessary for this observation. When two signal generators are adjusted

In frequencies very close to each other, your ear should be able to detect beats. Is the beat frequency equal to the difference between the two oscillator frequencies? Have your instructor verify your results.

SELF CHECK

1. Draw and label transverse waves and longitudinal waves such as those you produced in a slinky in Demonstrations section. Explain major differences and similarities between these two types of waves.

2. Define these terms:

   Propagating medium.
   Wavelength.
   Frequency.
   Period.
   Displacement.
   Amplitude.
   Phase.
   Standing wave.
   Constructive interference.
   Destructive interference.
   Beats.

3. a. Calculate the frequency for a light wave whose wavelength is 555 nanometers. Use 3×10^8 m/s as the speed of light.
   b. What is the wavelength of air of a sound whose frequency is 980 Hz. Consider the speed of sound in air to be equal to 1130 ft/s.

4. Relate a sine wave to a circle by drawing a sine-wave sketch having a reference circle as given in the text. Explain, in your own words, how the $2\pi f$ in the sine wave formula is related to circular motion.

5. Draw an accurate sine-wave sketch of current leading voltage by 60°.

6. Explain in detail the meaning of the "superposition principle."

7. Describe and give examples of waves in the following energy systems:

   a. Mechanical.
   b. Fluidal.
   c. Electrical.

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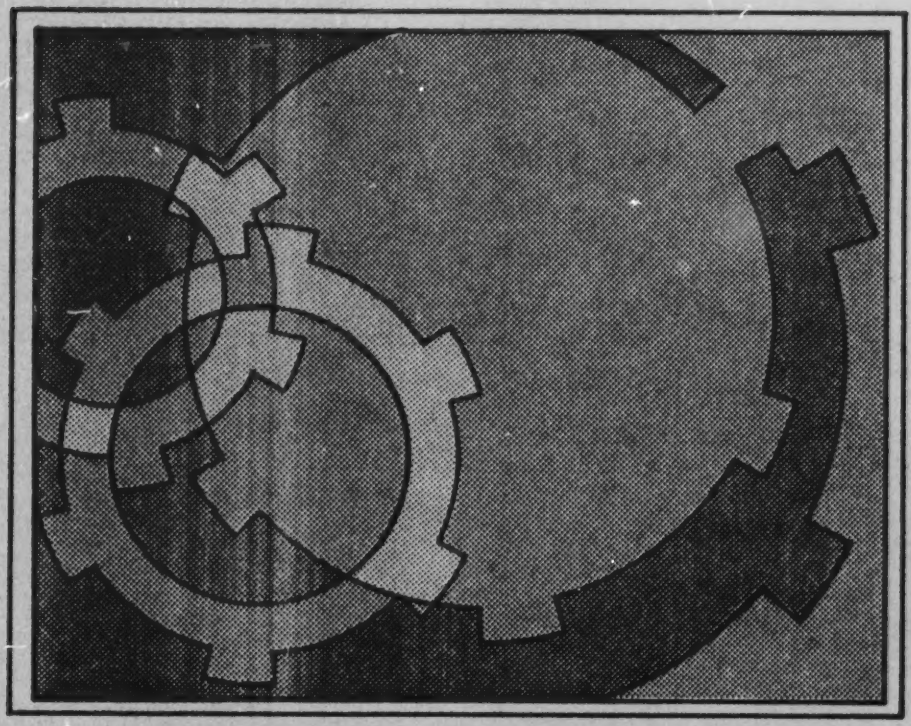
UNIFIED TECHNICAL CONCEPTS

MODULE 11 M 1

SPEED OF SOUND IN MATERIALS



MECHANICAL APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

Sound waves travel through various materials with a speed dependent upon the elastic properties of the materials. The velocity of sound generally is highest in solids, less high in liquids, and lowest in gases. In gases, the speed increases in proportion to the square root of the absolute temperature.

In the laboratory, the student will perform measurements on the velocity of sound in different media.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves."

OBJECTIVES

Upon completion of this module, the student should be able to:

- State the mathematical relationship that gives the dependence of the velocity of sound on material properties, for a gas, a solid, or a liquid.
- Calculate the speed of sound, given the atmospheric pressure and temperature.
- Determine the distance from the source to a receiver, given the time the sound takes, the temperature, and the atmospheric pressure.
- Given the necessary equipment, measure the velocity of sound in different materials.

DISCUSSION

Concept Module II-O, "Vibrations and Waves," described the propagation of sound waves through materials. Briefly, sound waves are longitudinal waves that involve successive compressions and rarefactions of the material through which the sound waves are propagating. The sonic wave moves at a speed called the "velocity of sound," which varies in different materials. This module discusses the velocity of sound in different types of materials.

Since sound waves involve periodic compressions and rarefactions of the material, the propagation of sound is affected by the elastic properties of the material. The elastic properties may be characterized for each material by a constant called the "modulus of elasticity." This modulus takes different forms, depending upon the nature of medium transmitting sound waves. It can be defined generally as the ratio of an applied pressure to the fractional change in shape produced by pressure. When pressure is exerted on a body, dimensions of the body change slightly. The elastic modulus relates the relevant change in dimensions of material to the applied force. Modulus of elasticity can be defined in its most general form by Equation 1.

$$E = \frac{\text{applied force per unit area}}{\text{fractional change in shape}} \qquad (1)$$

where

E = modulus of elasticity.

Since the denominator is dimensionless, the modulus of elasticity has units of force per unit area. The precise form of Equation 1 for gases, liquids, and solids will be given in the following pages.

The general equation for velocity of sound in different materials takes the form of Equation 2.

$$v = \left(\frac{E}{\rho}\right)^{\frac{1}{2}} \qquad (2)$$

where

v = velocity of sound.
E = elastic modulus.
ρ = density of material.

The velocity of sound is related to the elastic modulus. This equation is presented without proof as the defining equation for the velocity of sound in different media.

The correct elastic modulus in a gas is given by Equation 3.

$$E = \lambda p \tag{3}$$

where

p = pressure of gas.
λ = constant that depends upon atomic structure of gas.

For monatomic gases (helium, neon, argon, etc.), λ equals 1.67. For diatomic gases (oxygen, nitrogen, air, etc.), λ equals 1.4.

The pressure is related to other properties of a gas by the ideal gas law. Equation 4 states one form of the ideal gas law for pressure.

$$p = \frac{\rho RT}{M} \tag{4}$$

where

R = universal gas constant (8.317×10^7 erg/mole-K).
T = absolute temperature.
M = molecular weight of gas.

Equation 5 (a combination of Equations 2, 3, and 4) is a useful expression for the velocity of sound in a gas.

$$v_g = \left(\frac{\lambda RT}{M}\right)^{\frac{1}{2}} \tag{5}$$

where

v_g = velocity of sound in a gas.

The velocity of sound varies with the square root of the absolute temperature. The increase in speed of sound with increasing temperature can be understood easily. As the gas is heated, the neighboring gas molecules can collide with one another more easily and can propagate acoustic energy through the gas more rapidly.

Equation 5 also shows the effect of changing the type of gas. As the molecular weight of the gas decreases, the speed of sound in the gas increases. This increase is understandable because lighter molecules move more rapidly and transfer energy to their neighbors more easily. Thus, the velocity of sound in a gas such as helium will be relatively high because the molecular weight is low and, also, because the value of λ is higher.

Table 1 presents some values for the velocity of sound in various gases at 1 atmosphere pressure and 0°C or 100°C temperature. Examples A, B, and C show how the velocity of sound is calculated in various media.

TABLE 1. VELOCITY OF SOUND IN GASES

Gas	Velocity of Sound (m/s)
Air (0°C)	331.7
Air (100°C)	386
Argon	319
CO_2	259
He	965
Neon	435
N_2	334
O_2	316

EXAMPLE A: CALCULATION OF VELOCITY OF SOUND

Given: Assume that the air temperature is 0°C. You see a flash of lightning and hear the sound of thunder seven seconds after the lightning flash was visible.

Find: Distance to the flash. (Note: Thunderclaps are rare at 0°C, but they do occur.)

Solution: The velocity of sound in air at 0°C is 331.7 meters per second; therefore, the lightning flash was 331.7 x 7 = 2322 meters ≈ 2.3 km away.

As a general rule of thumb, sound travels in air at a velocity of 1/5 mile per second (about 1100 feet/second). Therefore, one may estimate distance of a lightning flash by counting the number of seconds between lightning flash and sound of thunder, and dividing by five. This gives distance in miles. This approach, of course, assumes velocity of light is so great that it can be neglected.

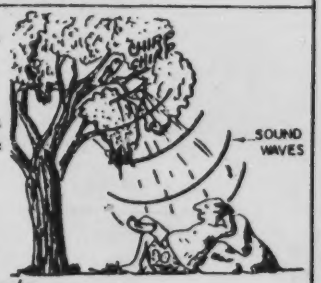
EXAMPLE B: CALCULATION OF VELOCITY OF SOUND

Given: Temperature of air is 40°C.

Find: Velocity of sound in air.

Solution: Velocity of sound increases with the square root of the absolute temperature. The value from Table I at 0°C (273 K) is used to obtain the following value:

$$(331.7)\left(\frac{273 + 40}{273}\right)^{\frac{1}{2}} = 355.2 \text{ m/s}$$



EXAMPLE C: CALCULATION OF VELOCITY OF SOUND

Given: Equation 5 for an ideal gas.

Find: Velocity of sound in oxygen at 0°C. Compare to the value in Table I.

Solution: The molecular weight of oxygen is 32. When the proper values are inserted into Equation 5, the following is the result:

$$v_g = \left(\frac{\lambda RT}{M}\right)^{\frac{1}{2}}$$

$$= \left[\frac{(1.4)(8.317 \times 10^7)(273)}{32}\right]^{\frac{1}{2}}$$

$$= 31{,}520 \ \frac{\text{cm}}{\text{s}}$$

$$v_g = 315.2 \text{ m/s}$$

Notice that the measured value, as given in Table I is 316 m/s. The difference arises because a real gas does not follow the ideal gas law exactly.

Now let us consider the speed of sound in a liquid. The basic equation for speed of sound is given by Equation 2. The proper elastic modulus to use in liquids is the bulk modulus given by Equation 6.

$$B = -\frac{p}{(\Delta V/V_o)} \qquad (6)$$

where

B = bulk modulus for liquids.
p = pressure.
ΔV = change in volume in a fluid element of original volume V_o subjected to pressure p (Figure 1).
V_o = original volume.

The minus sign is included in the definition because increasing pressure means decreasing volume. Thus, if p is positive, ΔV will be negative, and the minus sign ensures that the bulk modulus will be a positive quantity. The units of bulk modulus are force per unit area.

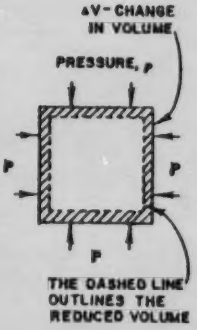


FIG. 1

According to Equation 2, the equation for the velocity of sound in a liquid then is Equation 7.

$$v = \left(\frac{B}{\rho}\right)^{\frac{1}{2}} \qquad (7)$$

where

v = velocity of sound in a liquid.

The values of elastic moduli of a liquid are higher than those of a gas, and the velocity of sound in a liquid are higher than that in a gas. Table 2 shows some values for various liquids, at 20°C. The velocity of sound in a liquid increases with temperature because the density ρ decreases. Specifically, for water the density decreases as the temperature increases above 4°C, and the bulk modulus also increases. (Water is anomalous, in that it expands again as the temperature drops below 4°C.) Thus, the speed of sound increases with temperature, but there is not a simple theoretical expression for the temperature dependence.

TABLE 2. VELOCITY OF SOUND IN LIQUIDS

Material	Velocity of Sound (m/s)
Fresh water	1481
Sea water	1514
Methyl alcohol	1130
Ethyl alcohol	1150
Mercury	1450
Turpentine	2150

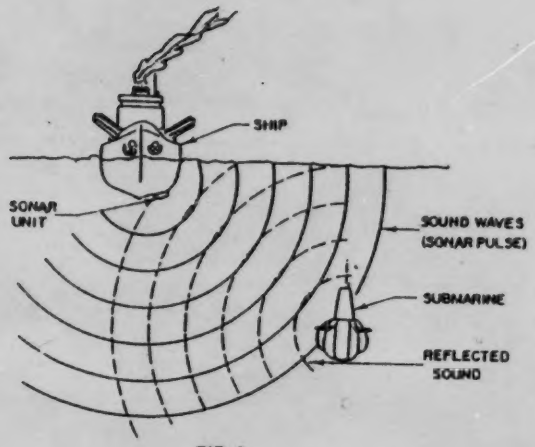


FIG. 2

The value of velocity of sound in sea water is of special interest because of sonar applications (Figure 2). This value depends upon many factors, including salinity of the water, depth below the surface, etc.; therefore, the value in Table 2 must be regarded only as an approximation (Example D).

EXAMPLE D: CALCULATION OF DISTANCE SOUND TRAVELS IN LIQUIDS

Given: A sonar operation (Figure 2) sends out a pulse of sound and receives an echo from a submarine 2 seconds later.

Find: How far away is the submarine?

Solution: By using the value of speed of sound in sea water from Table 2, the below value is obtained:

Distance = velocity x time/2

= (1514 x 2/2) meters

Distance = 1.514 km

Note: Factor 2 arises because the sound has traveled in a round trip from sonar set to the submarine and back.

The speed of sound in solids is more complicated. There are additional types of waves that can be propagated by solids because of the more rigid coupling between neighboring atoms. In addition to pure longitudinal waves, there may also be transverse waves, called "shear waves," propagated in a solid medium excited by an acoustic wave. Since the topic of speed of sound in solids can become quite complicated, we shall describe only the simple case of pure longitudinal waves.

In a long, thin bar of a solid (a sample that the length is much greater than the diameter of the bar) the relevant elastic modulus is Young's modulus. Young's modulus is often simply called the "modulus of elasticity of a solid." There are other elastic moduli of solids relevant to the transmission of shear waves. Young's modulus is defined by Equation 8.

$$Y = \frac{F/A}{\Delta \ell / \ell_o} \tag{8}$$

where

Y = Young's modulus.
F = tensile or compressive stress.
A = cross-sectional area.
$\Delta \ell$ = change in length of the sample of original length ℓ_o in response to the application of stress.
ℓ_o = original length.

Thus, Young's modulus gives the ratio of stress to the fractional strain produced by stress. If this relation is used in Equation 2, speed of sound in a solid bar is obtained from Equation 9.

$$v_s = \left(\frac{Y}{\rho}\right)^{\frac{1}{2}} \tag{9}$$

where

v_s = speed of sound in a solid.

Some tabulated values for the "bar" velocity of sound in solids are presented in Table 3. These values are higher than those in gases and in liquids.

TABLE 3. VALUES OF VELOCITY OF SOUND IN SOLIDS

Material	Velocity in Bar (cm/s)	Bulk Velocity (cm/s)
Aluminum	5.15×10^5	6.3×10^5
Brass	3.5×10^5	4.7×10^5
Copper	3.7×10^5	5.0×10^5
Steel	5.05×10^5	6.1×10^5
Glass (pyrex)	5.2×10^5	5.6×10^5
Rubber (hard)	1.45×10^5	2.4×10^5
Oak wood		4.0×10^5
Granite		3.95×10^5

For bulk samples of solids, the situation is more complicated and is beyond the scope of this module, although the student should note that for large bulk samples, the other elastic moduli also have to be considered. However, as a first approximation, Equation 7 can be used with the bulk modulus of the solid in calculating sound velocity. The velocity of sound in bulk material is higher than that in a bar sample. Values of bulk samples also are given (Table 3).

The values in Table 3 are relevant to a temperature of 20°C. The velocity of sound in solids may increase or decrease as temperature increases. This is a complicated phenomenon beyond the scope of this module.

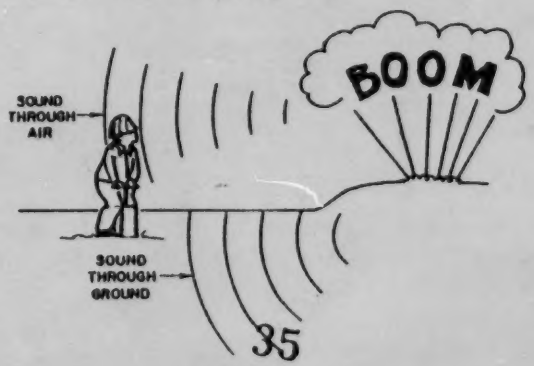
The values in Table 3 must be considered as approximations because they depend upon many factors, including material purity, composition, crystalline orientation (if appropriate), etc.

For an order-of-magnitude comparison between the velocity of sound in gases, liquids, and solids, observe typical values presented in Tables 1, 2, and 3. Note that velocity of sound in gases is lowest and that velocity of sound in solids is highest (Example E).

EXAMPLE E: CALCULATION OF SOUND VELOCITY IN AIR

Given: An observer is one-half mile away from the position of a dynamite blast.

Find: Time required for the sound wave transmitted through the granite rock to arrive at the observer's position and time required for a sound wave transmitted through air at 0°C to arrive. Compare the arrival time of a sound wave from a dynamite blast as it travels through granite rock and through air.



Solution: The sound must travel a distance of one-half mile, or 804.7 m. From Table 3, velocity of sound in granite is 3950 m/s. The time for sound to travel through granite is 804.7/3950 = 0.204 seconds. Velocity of sound in air is 331.7 m/s. The time for sound to travel through air is 804.7/331.7 = 2.43 s, over ten times as long as through rock.

SUMMARY

The velocity of sound waves is a function of the temperature and of the characteristics of the propagating medium. In gases, velocity is independent of pressure. In air at 0°C, velocity is 1120 feet (332 meters) per second, or approximately 760 miles per hour. Sound waves are transmitted by solids, liquids, and gases. The stiffer the material, the faster the sound waves travel through it because stiffness indicates that the particles of the material are coupled together tightly and, therefore, are more responsive to particle motion.

VELOCITY MEASUREMENT OF SOUND IN AIR

MATERIALS

One-foot piece of metal pipe (gong).
One-foot metal bar (striker).
Oscilloscope.
Microphone.
Meter stick.
Connecting wire (20 m).
6-volt battery.

PROCEDURE

1. Set up experimental apparatus as illustrated in Figure 3.

2. Strike pipe with bar while observing oscilloscope trace. Adjust oscilloscope trigger level to produce a single trace when electrical contact is made between bar and pipe. Adjust oscilloscope time base and sensitivity to display audio signal.

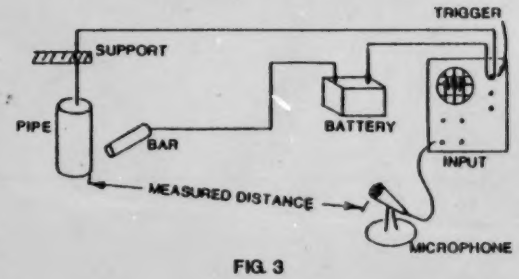


FIG. 3

3. Strike pipe with bar, and measure time between beginning of oscilloscope trace and beginning of audio pulse detected by microphone. Record in Data Table 1.

4. Measure distance from pipe to microphone, and record in Data Table 1.

5. Calculate velocity of sound in air.

6. Measure and record air temperature.

7. Calculate and record speed of sound in air, using the method illustrated in Example B.

8. Compare the two values for speed of sound in air and explain why they are not the same.

VELOCITY MEASUREMENT OF SOUND IN A METAL ROD

MATERIALS

Metal bar with two holes (Figure 4); metal should be one of those from Table 4.
Acoustic pickups; for example, accelerometer type available from Vibration Measurement Corporation.
Hammer.
Grease (as required).
Electric wiring (as required).

PROCEDURE

1. Set up apparatus (Figure 4). The acoustic pickups are inserted in holes drilled in the metal bar at various positions.

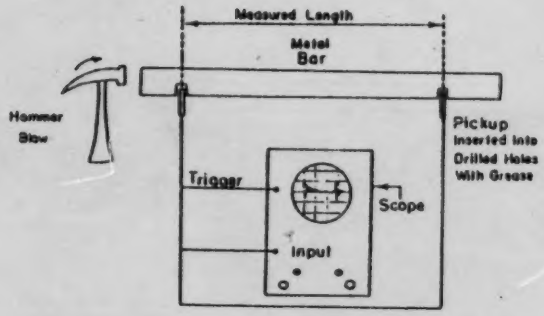


FIG. 4

Some acoustic pickups are threaded. In that case, holes in the bar should be threaded and pickups screwed in. Otherwise, pickups should fit snugly in holes. Grease should be used to ensure that sound will be transmitted from bar to pickups.

2. Strike end of rod with hammer. This will excite an acoustic wave that will travel down rod. The oscilloscope is triggered by the arrival of sound pulse at first pickup. The time interval between the arrival of pulse at first sensor and at second sensor can be measured on the oscilloscope face. Record in Data Table 2.

Sweep speed may have to be adjusted so that the pulse from the second pickup will fall on the oscilloscope trace. Repeatedly strike the end of bar until scope triggers reliably and you can make the measurement.

3. Measure distance between the two sensors, and calculate the velocity of sound. Use the known value of Young's modulus for rod material (Table 4) and use Equation 9 to calculate velocity of sound in the rod. Compare the two values and explain any difference in values.

TABLE 4. YOUNG'S MODULUS FOR METALS	
Metal	Young's Modulus
Aluminum (commercially pure)	6.9×10^{11}
Brass (yellow, 65% Cu, 35% Zn)	10.3×10^{11}
Copper (commercially pure)	12.8×10^{11}
Stainless steel (17-20% Cr, 8-11% Ni)	19.3×10^{11}
Titanium (commercially pure)	11.6×10^{11}
Tungsten (commercially pure)	34.5×10^{11}

DATA TABLE 1: VELOCITY MEASUREMENT OF SOUND IN AIR			
Time of Travel of Audio Pulse	Distance d Traveled	Velocity of Sound (Experimental)	Velocity of Sound (Calculated)

DATA TABLE 2: VELOCITY MEASUREMENT OF SOUND IN METAL				
Kind of Metal	Young's Modulus (Table 4)	Length Between Sensors	Time Between Pulses	Velocity of Sound

SELF CHECK

1. State a mathematical relation that gives the dependence of velocity of sound upon material properties for the following:

   Gas.
   Liquid.
   Solid bar.

2. What is the velocity of sound in air at 1 atmosphere pressure and a temperature of 57°C?

3. When you are in the ocean, you see the flash of light from an explosion and hear the sound of the explosion arrive through the water 3 seconds later. How far away was the explosion? If the air temperature is 0°C, how much later will the sound arrive through the air?

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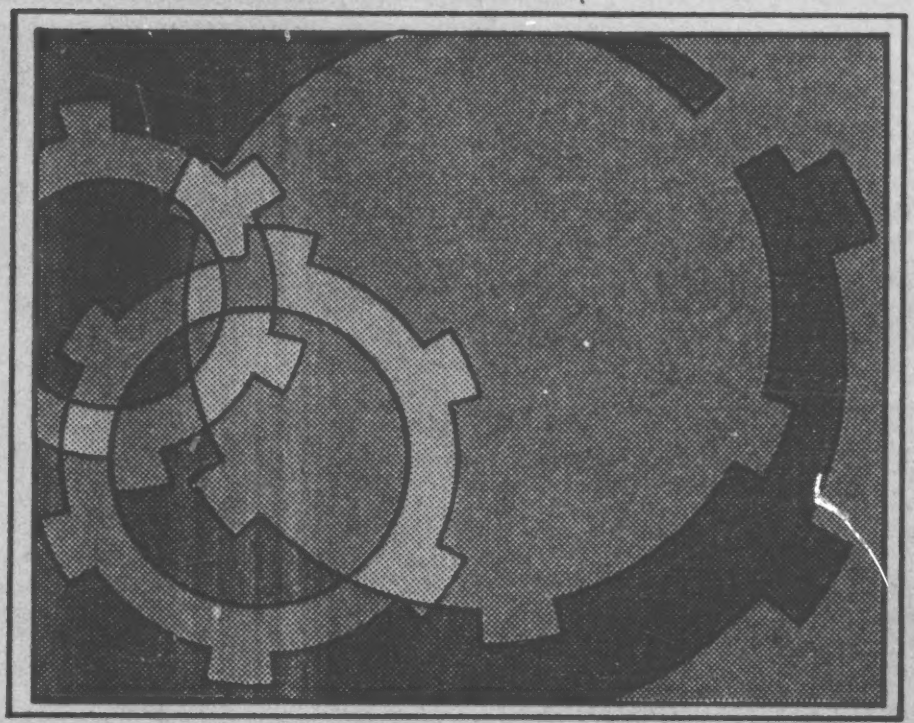
UNIFIED TECHNICAL CONCEPTS

MODULE 11 M 2

REFLECTION AND ABSORPTION OF SOUND ENERGY



MECHANICAL APPLICATION OF

VIBRATIONS AND WAVES

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

When a sound wave strikes a solid surface, part of the sound energy is reflected from the surface. This produces the phenomenon of an echo. The remainder of the energy penetrates into the material and is absorbed. Hard, rigid materials reflect a large fraction of incident energy. Softer, porous materials absorb a large fraction. Absorbing materials, such as acoustic tile, are used to reduce noise levels in homes, offices and industry.

The student will learn to express sound levels quantitatively in terms of the decibel unit. He will learn to make quantitative measurements of sound level reflected from surfaces of various materials. He will learn to measure a reduction in sound level when various types of material are used as sound absorbers.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves," and have the ability to solve problems involving logarithms.

OBJECTIVES

After completing this module, the student should be able to:

- Define:

  Decibel.
  Absorption coefficient for sound.
  Reflection coefficient for sound.

- Give the formula relating the absorption and reflection coefficients for sound and define each term.

- Given necessary sound absorption information for a given material, calculate the amount of sound energy reflected from a solid surface.

- Calculate reduction of sound level (in decibels) for an absorbing material, given appropriate information.
- List four absorbing materials useful in reducing noise.
- Given the required equipment, measure sound levels transmitted through various materials.
- Given the required apparatus, measure sound levels reflected from various materials.

DISCUSSION

Module II-0 described sound as a wave that travels through the atmosphere. Consider what happens when sound waves encounter a solid obstacle. Part of the energy carried by the sound wave may be reflected (Figure 1). The sound wave is shown striking the surface at an angle i measured from the normal to the surface. The reflected wave emerges at an angle r to the normal. For "specular" reflection from a smooth, flat surface the angles i and r are equal. Thus, the sound wave is reflected from a plane surface with an angle of reflection equal to the angle of incidence. This is the same law that holds for reflection of light from a plane mirror. Reflection of sound energy is a familiar phenomenon; it is the process that produces echoes.

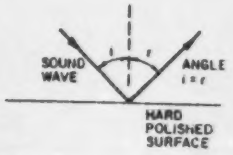


Fig. 1

All of the energy incident on the surface is generally not reflected. Some of this energy may be transmitted into the material; this energy will be absorbed. The process of absorption of sound energy is sketched in Figure 2. Thus, reflected sound waves will always have less intensity than incident waves. Use of acoustic absorbing materials is an important method for removing sound energy from the environment to reduce noise levels.

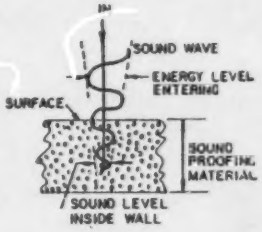


Fig. 2

Because noise control is important, the reflection and absorption of sound energy is a significant topic. This application module will describe these phenomena in association with the properties of materials used for acoustic applications.

SOUND LEVEL

The intensity level L of a sound wave that delivers I ergs of sound energy per square centimeter per second is given by:

$$L = 10 \log_{10} I/I_0. \qquad (1)$$

In this equation I_0 is a reference intensity level, taken to be 10^{-9} erg/cm^2-s. The choice of this value is somewhat arbitrary, but is a convenient level because it corresponds roughly to the minimum flow of sound energy that can be detected by a human ear. Intensity level is expressed in units of decibels (dB).

Equation 1 shows that sound levels involve a ratio of sound energy to a reference level. There are several reasons for expressing sound levels in this way. Use of the ratio is realistic because the magnitude of a sensation as perceived by the human ear turns out to be roughly proportional to the logarithm of the intensity rather than to the absolute value of the intensity itself--at least for pure tones. Also, the use of logarithms is convenient because the human ear can detect changes in sound over a very large range. Logarithmic scales compress this large range into a scale of numbers of convenient size.

Table 1 presents values of sound level in decibels for various conditions. When sound energy flow I is 10^{-9} erg/cm^2-s, $I = I_0$ and:

$$L = 10 \log I = 0$$

so that zero of the scale is equal to the threshold of hearing.

TABLE 1. TYPICAL EVERYDAY SOUND ENERGY

Description of Sound	Sound Level (dB)
Threshold of hearing	0
Rustle of leaves	10
Whisper	20
Library	40
Ordinary conversation (at 3 feet)	60
City traffic	75
Subway	90
Riveting hammers	95
"Rock" band	110
Threshold of pain	120 - 140

Examples A and B show how Equation 1 is used:

EXAMPLE A: FLOW OF SOUND ENERGY

Given: Sound at 35 dB.

Find: Flow of sound energy to which this corresponds.

Solution: $35 = 10 \log I/10^{-9}$

$I = 10^{-9}$ antilog $35/10 = 3162 \times 10^{-9} = 3.162 \times 10^{-6}$ erg/cm^2-s.

Decibel notation is sometimes used to describe relative intensities of two sound levels, without direct reference to an absolute level. In this case, if sound energy per unit area per unit time is denoted I_1 and I_2 for two sound waves, the relative intensity of the first sound compared to the second (in decibels) is given by:

EXAMPLE B: CHANGE IN SOUND ENERGY

Given: The minimum perceptible change in sound level that the human ear can detect is about 3 dB.

Find: How large a change in sound energy this represents.

Solution: Represent the two sound levels by I_1 and I_2 (with I_1 being chosen as the larger).

$L_1 = 10 \log_{10} I_1/I_0$ and $L_2 = 10 \log_{10} I_2/I$.
$L_1 - L_2 = 3$ dB

Then:

$$10 (\log (I_1/I_0) - (\log I_2/I_0)) = 3$$

From the properties of logarithms:

$$10 \log \left(\frac{I_1}{I_0} / \frac{I_2}{I_0}\right) = 10 \log I_1/I_2 = 3$$

I_1/I_2 = antilog $3/10 = 1.995$, say 2.0.

Thus energy flow must be about twice as large for one sound level as for the other, before the human ear can detect a change.

$$L = 10 \log_{10} I_1/I_2. \qquad (2)$$

See Example C.

EXAMPLE C: RELATIVE SOUND INTENSITIES

Given: Sound level 1 is 35 dB below sound level 2.

Find: Relative sound intensities.

Solution: L = -35 dB. Using Equation 2,

$$10 \log_{10} I_1/I_2 = -35$$

$$I_1/I_2 = \text{anilog } -35/10 = 0.000316.$$

Thus sound wave 1 is only 3.16×10^{-4} times as intense as sound wave 2.

REFLECTION COEFFICIENT

Consider how much sound energy is reflected when a sound wave strikes a solid surface. The fraction reflected is related to velocity of sound described in Module 11M1.

If a sound wave traveling in air is reflected from a flat, smooth, solid surface, the fraction of sound energy reflected will depend on the velocity of sound in the two materials.

For rigid, nonporous surfaces like steel, glass or sealed concrete, the action of a sound wave on the surfaces is a compressive action. Vibrations in the air are coupled into the solid through acoustic surface pressure. Waves of the same frequency are then driven into the solid.

The fraction R of acoustic energy reflected from a surface is call the reflection coefficient. For some practical cases of interest, the value of the reflection coefficient will be given by:

$$R = (\rho_2 c_2 - \rho_1 c_1)/(\rho_2 c_2 + \rho_1 c_1) \qquad (3)$$

where

ρ_1 = density of air.

ρ_2 = density of solid.

c_1 = velocity of sound in air.

c_2 = velocity of air in solid.

For a typical case, the density and velocity of sound in a solid are both much higher than that in air. The value of R given by Equation 3 approaches unity. Most sound energy will be reflected from a hard, rigid, nonporous surface. Equation 3 is strictly applicable only when the sound wave is incident perpendicularly to the surface.

For porous solid materials like unglazed bricks, cloth and acoustic tile, acoustic pressure on the surface not only compresses the solid, but also moves air in the voids within the solid. This tends to absorb sound energy and dampen the sound wave. The action of the trapped air pockets in the porous surface causes a smaller reflection of sound energy than from a smooth rigid surface. This has an analogy with thermal conduction properties, since materials having air spaces trapped in them tend also to be good thermal insulators. See Example D.

EXAMPLE D: REDUCTION IN SOUND WAVES

Given: 50% of the energy in a sound wave is reflected from a wall.

Find: Reduction of the sound level in decibels.

Solution: Let I_1 be reflected sound level and I_2 the incident sound level. $I_1/I_2 = 0.5$. Using Equation 2:

$$L = 10 \log_{10} 0.5 = -3.01 \text{ dB}.$$

Thus the sound level is decreased by 3 dB.

ABSORPTION COEFFICIENT

When sound energy strikes a massive solid material, part of the sound energy is reflected as discussed above. The rest of the energy will be transferred into the material, where it will eventually be absorbed. Figure 2 has indicated the gradual absorption of sound energy as it passes through the material. If the material is thick enough, all sound energy will be absorbed. If the material is not thick enough, some of the sound energy will be transmitted through the material and will emerge on the opposite side. Thus, one can hear voices in another room if the wall between the rooms is thin.

The fraction of sound energy that is absorbed when sound is incident on a material is called the absorption coefficient A. Absorption coefficient is a number between 0 and 1. It is related to the reflection coefficient R (see Equation 3) by the equation:

$$A = 1 - R - T \qquad (4)$$

where T is the fraction of sound energy transmitted through the material. As the material becomes thicker, T will approach zero, and all sound energy will be either reflected or absorbed. Materials that are relatively soft and porous tend to have relatively high values of A. Materials that are hard and rigid have relatively low values of A.

An important consideration is the relation of absorption coefficient to noise suppression and the acoustic properties of rooms. For example, if an auditorium has very highly absorbing walls, sound will die out rapidly and a speaker's voice will seem dead in quality. On the other hand, if there is too much reflection from the surfaces in the room, sound waves will not die out rapidly enough; there will be too much reverberation. These echoes will make the sound difficult to understand. The topic of reverberation time is treated in Module 11F4. For now, simply note that knowledge of the acoustic properties in rooms can be an important topic.

Table 2 presents some values for absorption coefficients for various materials. A high value of absorption coefficient means that most of the energy is absorbed. The specific value of absorption coefficient depends on how the material is mounted, size of the tested material and the test chamber. Therefore, the values presented in Table 2 should be regarded only as approximations. Nonetheless, the relative effectiveness of acoustic absorbing materials and of porous materials in absorbing sound energy is apparent from the Table.

TABLE 2. ABSORPTION COEFFICIENTS			
Material	Acoustic Frequency (Hertz)		
	125	500	2000
Acoustic paneling	0.16	0.50	0.80
Brick wall, painted	0.012	0.017	0.023
Brick wall, unpainted	0.02	0.03	0.05
Draperies, heavy velour	0.14	0.55	0.70
Fiberglass tile	0.04	0.63	0.83
Floor, concrete	0.01	0.02	0.02
Floor, wood	0.06	0.06	0.06
Floor, carpeted	0.11	0.37	0.27
Glass	0.04	0.05	0.05
Marble	0.01	0.01	0.02
Pine paneling	0.10	0.10	0.08
Wall, concrete	0.01	0.02	0.02

Hard and rigid materials like glass absorb only a few percent of the acoustic energy and reflect the rest. Porous materials like acoustic paneling can absorb much of the acoustic energy. Note: Absorption generally increases as acoustic frequency increases, as Table 2 also indicates.

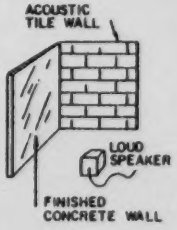
Table 2 also shows how sound levels may be reduced. Bare walls, floors and windows have low absorption coefficients and reflect most sound energy incident on them. Use of materials such as acoustic paneling, carpeting and draperies will increase absorption of sound energy, and thus will lead to a quieter environment. See Example E.

EXAMPLE E: REFLECTED SOUND CALCULATIONS

Given: Fraction of sound energy absorbed by a concrete wall before and after acoustic paneling is used to cover the wall.

Find: The percentage of sound energy reflected at a frequency of 500 hertz.

Solution: From Table 2, the bare concrete wall absorbs only 2% of the incident energy at 500 hertz, and hence reflects 98%. If it were covered with acoustic paneling, it would absorb 50% and reflect 50%.



Sound-absorbing materials have many important practical implications. They are used to reduce noise levels in offices, factories and homes. They are used selectively in concert halls and auditoriums to reduce unwanted reflections of sound. Sound barriers are erected along the edges of busy highways, freeways and airports.

Control of noise levels thus has many important consequences. Performance of workers decreases as noise levels increase above certain levels. Use of acoustic tile has become a familiar means of providing quiet work environments.

NOISE LEVEL EXPOSURE

Standards have been adopted for noise levels by many governmental and industrial regulations. Many such laws and regulations have reference to a so-called "A-scale" of relative weighting for sound level as a function of frequency. The A weighting curve is shown in Figure 3. It is roughly matched to human auditory sensitivity. The expression of a noise level using the A weighting scale is in units of dBA. To determine sound level using this weighting scale, the frequency composition of the sound must be determined. At each frequency, the sound level is multiplied by a factor from Figure 3 appropriate to that frequency. The result is a single number, in dBA, that expresses the noise level. The use of dBA for measurement of noise level provides a better expression of response of the human ear to that noise than the simpler unit if dB, which only indicates the total flow of acoustic energy, without regard to frequency.

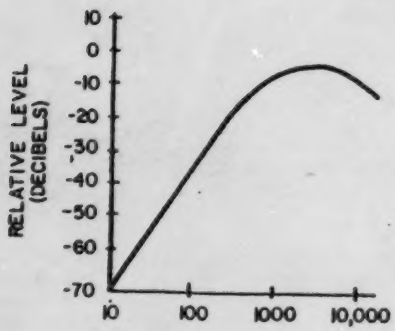


Fig. 3

As one example of a standard for noise exposure based on A weighting, the numbers in Table 3 express maximum permissible

exposure of industrial workers to noise levels, for various durations of exposure per day. As time of exposure increases, the maximum permissible exposure, in dBA, decreases. These are values set by the Occupational Safety and Health Administration (OSHA).

TABLE 3. MAXIMUM PERMISSIBLE EXPOSURE TO NOISE (WITH WEIGHTED "A" RESPONSE)

Duration of Exposure per Day (Hours)	Sound Level (Decibels, "A" Response)
8	90
6	92
4	95
3	97
2	100
1 1/2	102
1	105
1/2	110
1/4 or less	115

In addition, many other Federal agencies, many individual states and many cities have laws and regulations limiting noise levels for a wide variety of conditions.

Finally, consider the sound level meters, which are commonly used for the measurement of sound energy levels. They are used for monitoring noise levels in industry and for enforcement of laws and regulations concerning noise levels.

Sound level meters have a variety of possible frequency weighting characteristics. If a meter has a so-called flat response, it has no weighting. It responds equally well to sound at any frequency. Many meters have scales called A, B and C. The A weighting has been described above. The B weighting provides moderate reduction at low frequencies. The C weighting provides almost flat response, with a small amount of reduction at low frequency.

A block diagram of a typical commercial sound level meter is shown in Figure 4. The operator may switch between the different weightings as desired. Sound level is simply read from the meter.

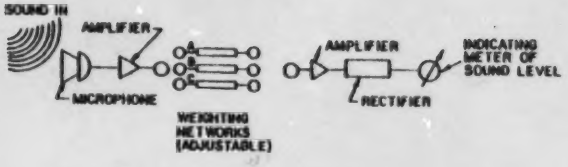

SOUND IN
AMPLIFIER
MICROPHONE
A
B
C
WEIGHTING NETWORKS (ADJUSTABLE)
AMPLIFIER
RECTIFIER
INDICATING METER OF SOUND LEVEL

Fig. 4

DEMONSTRATION OF REFLECTION OF SOUND FROM THE WALL OF A ROOM

MATERIALS

Sound level meter. (A sound level meter with a flat response could be used, if available. Most commercial sound level meters have weighted responses, as described in the text; a meter with A, B and C responses is also suitable. Model 1565-B available from Gen Rad, formerly General Radio, or equivalent, is suitable. If no sound level meter is available, a microphone and oscilloscope may be substituted.)
Electric buzzer.
Battery.
Switch.
Box made of acoustic tile.
Test materials: sheets of acoustic tile, wood, steel, glass and other materials, if desired.
Speaker.
Signal generator.
Electric wiring (as required).

PROCEDURES

1. Set up equipment as indicated in Figure 5. If a sound level meter is not available, use microphone and oscilloscope instead. This alternative arrangement is shown in Figure 6. Output of microphone is used as input to y-axis of the oscilloscope. Oscilloscope should be free-running, that is, internally triggered.

   Acoustic tile (1) in front of the buzzer creates a sound shadow, so that most sound reaching the sound level meter (or microphone) is reflected from the wall. The

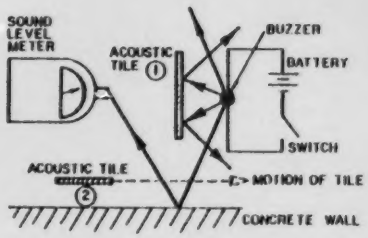


Fig. 5

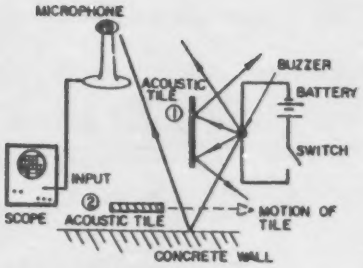


Fig. 6

experiment will work best if the wall is a hard, rigid wall, for example concrete blocks.

2. Turn on buzzer. Observe reading of sound level meter. (If meter is one with A, B and C responses, use C response.) If using microphone and oscilloscope, observe vertical deflection of oscilloscope trace.

3. Move a piece of acoustic tile (2) through various positions along the wall, as indicated in Figure 5 (or Figure 6). When acoustic tile crosses path of the sound that is reflected from wall to meter, the meter reading will reach a minimum.

Observe reading of meter as tile is moved to various positions; note decrease in meter reading (or in signal on oscilloscope trace).

MEASUREMENT OF SOUND LEVEL TRANSMITTED THROUGH VARIOUS

MATERIALS

Same materials as first laboratory experiment.

PROCEDURES

1. Set up apparatus as shown in Figure 7. If no sound level meter is available, use a microphone and oscilloscope instead. Oscilloscope should be free running, with output of

microphone used as input to the y-axis.

2. Activate buzzer and measure reading of sound level meter (in decibels). Use C response of meter, if appropriate.

3. Insert box covers of various materials over opening of box. Use covers of acoustic tile, glass, wood, steel, and other materials, if appropriate. In each case, measure reading of sound level meter (or vertical deflection on oscilloscope trace).

4. Arrange materials in order of sound transmission. Comment on the ordering, with reference to what you know about properties of the different materials.

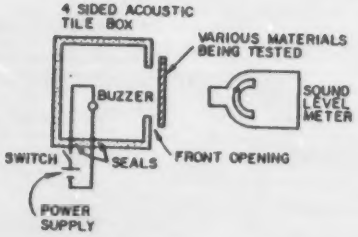


Fig. 7

MEASUREMENT OF SOUND LEVEL TRANSMITTED THROUGH VARIOUS MATERIALS

MATERIALS

Same materials as first laboratory experiment.

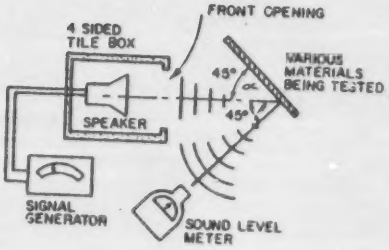


Fig. 8

PROCEDURES

1. Set up apparatus as shown in Figure 8. Signal generator is used to drive speaker at a fixed frequency. Again, use C response of meter, if appropriate. Sound level meter may be replaced by microphone and oscilloscope, with oscilloscope free-running and output of microphone used as input to y-axis of oscilloscope.

2. Use a steel plate as material being tested. Measure reading of sound level meter (in decibels) or vertical deflection on oscilloscope trace.

3. Replace steel plate with several other materials (e.g., glass,

wood, acoustic tile and other materials) if available. Keep angles α and β, indicated in Figure 8, the same for each material.

4. Arrange different materials in order of intensity of sound reflected from surface. Compare ordering to order obtained in second experiment. Comment on ordering, using your knowledge of acoustic properties of materials.

5. Change frequency of signal generator. Repeat steps 2 and 3. Compare results obtained at second frequency to those obtained at first frequency. The two frequencies should be considerably different (e.g., 1000 Hz for the first frequency and 125 Hz for second). Compare the results with those obtained in steps 2 and 3.

SELF CHECK

1. Define:

   Decibel.
   Absorption coefficient.
   Reflection coefficient for sound.

2. State the relationship between absorption and reflection coefficients for sound.

3. A very thick wall of concrete blocks has an absorption coefficient of 0.30. A sound wave striking the wall delivers 10^{-10} watts/cm^2 to the wall. What is the sound level in the reflected sound wave?

4. A sound wave striking a wall of acoustic tile delivers 10^{-2} erg/cm^2-s. The reflected sound level is 2.5×10^{-3} erg/cm^2-s. What is the reduction in dB due to absorption by the tile?

5. List four absorbing materials useful in reducing noise.

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See especially Chapter 18.

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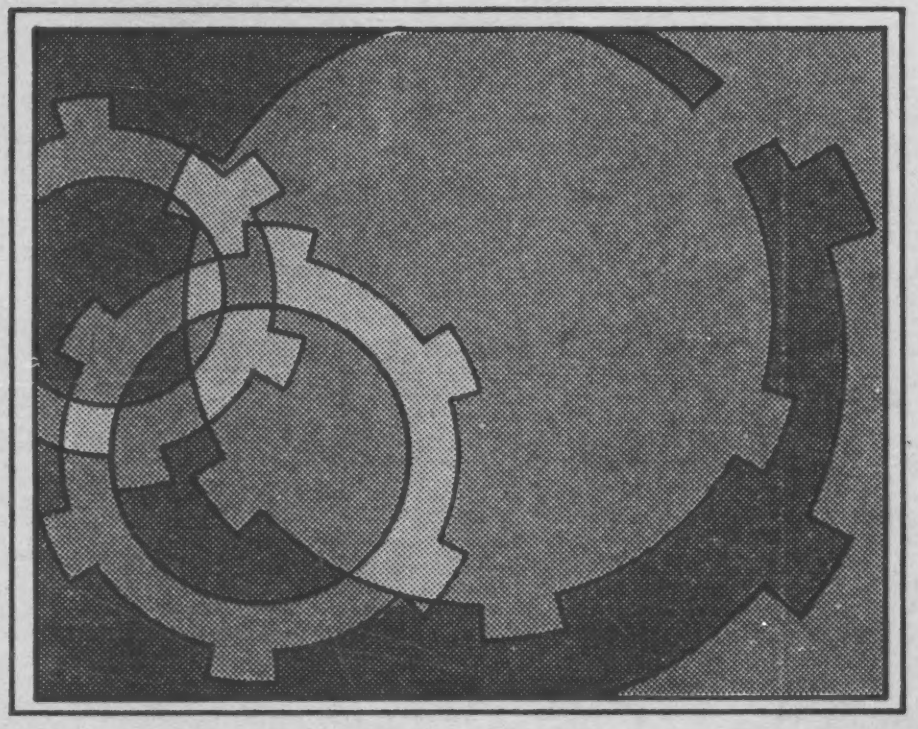
UNIFIED TECHNICAL CONCEPTS

MODULE 11 M 3

VIBRATION ISOLATION



MECHANICAL APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

Concept Module II-0, "Vibrations and Waves," explains how energy is transferred from one place to another via waves. In some systems, every attempt is made to maximize energy transfer via waves. For example, stereo sets and musical instruments are designed to provide maximum transfer to wave energy to the listener. On the other hand, mechanical waves or vibrations may be undesirable in some instances, and attempts are made to minimize their effects.

Mechanical vibrations are produced by almost every kind of machinery. Electric motors, internal combustion engines, air compressors, vacuum pumps, printing and manufacturing presses, and fans all produce unwanted vibrations. If these vibrations are not eliminated or reduced to an acceptable level, they can destroy the machinery or limit its proper operation.

In this module the characteristics of steady-state vibrations will be illustrated, and the methods used to eliminate or reduce these vibrations in various kinds of machinery will be discussed. In the laboratory, the student will measure the acceleration of a vibrating system and its support to determine transmission of vibration.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibriations and Waves."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Distinguish between vibrations produced by internal and external sources. Give at least two examples of each type, including a labeled drawing of each.
- Define the following terms:

Vibration.
Periodic.
Amplitude.
Transient vibration.
Shock.
Isolation.
Steady-state vibrations.
Superposition.
Vibrators.
Frequency.
Period.

- Calculate transmission of mechanical vibrations, given acceleration values of machinery and support structure.

- Given the appropriate equipment, measure the accelerations of a vibrating system and its support, and determine the transmission of the isolating system.

DEFINITIONS

In Concept Module II-0, "Vibrations and Waves," the word "vibration" was used to describe waves or wavelike motion. In this module, the broader meaning of vibration will be used. Vibration indicates any to-and-fro motion or oscillation of a mechanical system.

Some devices are specifically designed to cause vibration or shaking and are called vibrators. Foundries use special vibrators to loosen castings from molds. Vibrators also are used in medicine to stimulate blood circulation and muscles. All sound begins with a vibrating object; for example, when a person speaks, the vocal chords initiate the vibration that we recognize as speech sounds.

On the other hand, vibrations that weaken structures and produce wear and tear in industrial machines certainly are undesirable. In cities, trucks, subways, elevated trains, and other traffic cause considerable vibrations that are transmitted effectively throughout the steel structure of a large building. Engineers deal with this problem by using special vibration-absorbing materials. Pneumatic tires are used on automobiles in an effect to reduce undesirable vibrations created by running over rough surfaces. Automobile engines are mounted on rubber supports to reduce vibration; and more cylinders have been added to engines to smooth out engine vibration, resulting in a smoother flow of power.

Many industries deal with the problem of vibration and its effect on machinery. High-speed, rotating parts produce vibration unless they are accurately balanced. The supports of such rotating parts were once bolted securely to the machine frame or its foundation. Today the supports for high-speed, rotating parts are mounted on flexible connections, such as springs or rubber, thereby reducing the effect of vibration in machines.

Mechanical vibrations have various frequencies and amplitudes. Frequency is the number of complete vibrations made during a certain time period. Amplitude is the maximum distance the item travels from its normal position of equilibrium or rest. The period is the time required to make one complete vibration or oscillation, and is related to frequency by Equation 1.

$$T = \frac{1}{f} \qquad (1)$$

where

T = period (time) in seconds.
f = frequency in Hz.

Vibrations that repeat themselves at regular intervals are

called periodic. Both amplitude and period are shown in Figure 1 for a vibration that varies as a sine wave.

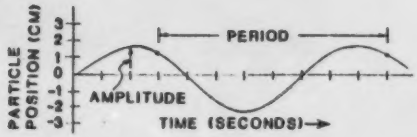


FIG. 1

Most vibrations are complex, meaning that they are the sum of two or more simple vibrations. Figure 2 illustrates the adding of vibrations 1 and 2 to obtain the resultant, or composite, vibration 3.

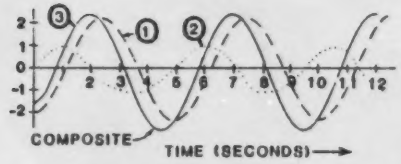


FIG. 2

Figure 3 illustrates the nature of other types of complex vibrations.

If machinery vibrates in a regular manner, the vibrations are called "steady-state." These vibrations normally are found in all machines and appliances, such as washers, dryers, and electric motors. Vibrations that are sudden and die out quickly are called "transient vibrations," or "shock vibrations." Figure 4 illustrates transient and steady-state vibrations.

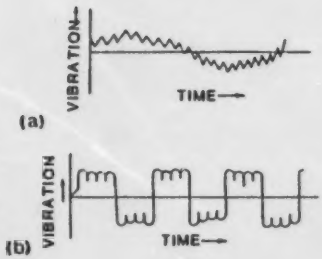


FIG. 3

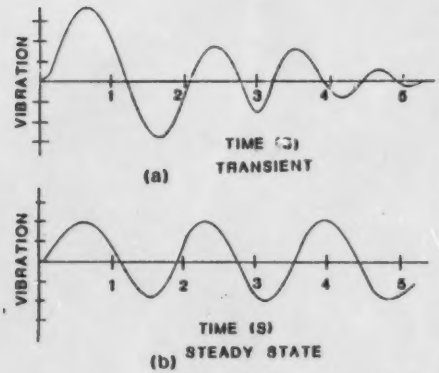


FIG. 4

TWO CLASSES OF VIBRATIONS

EXTERNAL

Vibrations in this class are caused by sources external to the machinery or equipment. Three examples of this class are illustrated in Figures 5, 6 and 7.

Highly variable vibration of wheels running over an uneven road surface are reduced greatly by automobile springs and shock systems. The ride inside the car is relatively smooth and comfortable.

A delicate electronic instrument is mounted on springs to absorb vibrations produced by the motion of the bulkhead to which it is attached.

FIG. 5

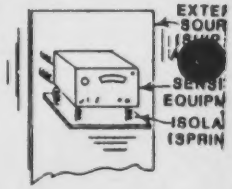


FIG. 6

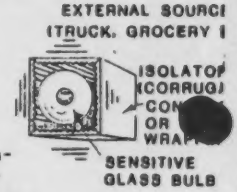


FIG. 7

A light globe is protected from external vibrations by a corrugated container or wrapper in which it is packaged.

INTERNAL

Vibrations in this class are produced by the machine's moving parts, which cause the entire system or machine to shake. Three examples of this class of vibration are shown in Figures 8, 9 and 10.

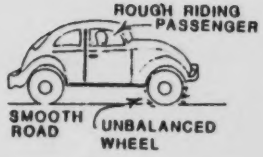


FIG. 8

An unbalanced automobile tire (Figure 8) can cause vibration in the entire car. This vibration can be reduced greatly when the sheel is balanced.

Vibratory forces frequently are induced in electrical machinery by magnetic effects. These effects can be reduced if slots are placed in the armature lamination to reduce jerking caused by the rapidly-changing magnetic field (Figure 9).

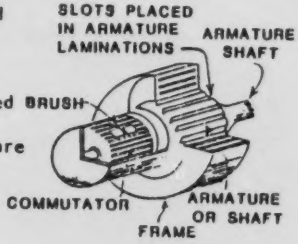


FIG. 9

An automobile axle bearing that has become worn and does not have even tolerances (Figure 10) will cause the wheel to vibrate, particularly at high speeds. This kind of vibration usually is reduced only by replacement of the worn bearings.

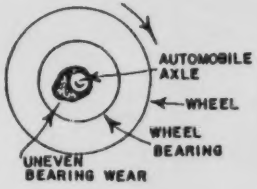


FIG. 10

VIBRATION ISOLATION

Excessive vibration must be isolated or reduced for the protection of personnel and of the equipment itself. Effectiveness of vibration isolation provided by a system may be evaluated by measuring the amount of vibration transmitted through an isolating medium.

Transmission of a vibration-isolation system can be measured by determining accelerations on both support structures and vibrating machinery. Transmission of the system for external vibration is given by Equation 2.

$$T_E = \frac{a_m}{a_s} \qquad (2)$$

where

T_E = external transmission of a vibration isolation system.
a_m = acceleration of machinery.
a_s = acceleration of support.

For internal vibrations, transmission is defined by Equation 3.

$$T_I = \frac{a_s}{a_m} \qquad (3)$$

where

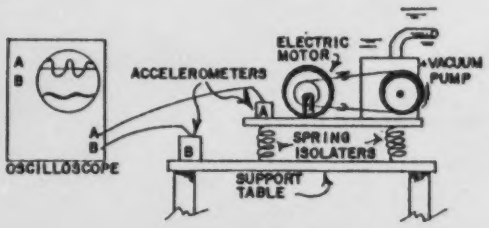
T_I = internal transmission of a vibration-isolation system.

In either case, acceleration of the vibrating source is placed in the denominator, and acceleration of the isolated structure or machinery is placed in the numerator. The smaller acceleration is always the numberator.

Example A illustrates use of Equation 3 to solve a problem.

EXAMPLE A: TRANSMISSION CALCULATION

Given: A vacuum pump is mounted on springs (see sketch). Amplitude of trace A is 2 cm peak-to-peak, and peak-to-peak amplitude of trace B is 0.2 cm. (Note: The amplitude on the oscilloscope of trace A or Trace B is proportional to the acceleration of the pump and table, respectively.)



Find: T_I.

Solution: Transmission of the spring isolators is

$$T_I = \frac{a_s}{a_m} = \frac{\text{acceleration of support}}{\text{acceleration of vacuum pump}}$$

$$= \frac{0.2 \text{ cm}}{2 \text{ cm}}$$

$$T_I = 0.1$$

Springs reduce vibrations to 10% of their original value.

SUMMARY

Vibration in a mechanical system is a wavelike, to-and-fro motion. When such vibration is undesired, it can be isolated by use of mounts that absorb some, or hopefully most, of it. The difference between the internal vibration of the equipment and the external vibration of its support can be measured to calculate the effectiveness of the mounts. The transmission of vibration refers to the amount of vibration "transmitted" from source to support.

VIBRATION ISOLATION AND TRANSMISSION

MATERIALS

Accelerometer with cable. (Two accelerometers are preferable for this experiment.)
Aluminum block 1" x 2" drilled and tapped to accept accelerometer. (If two accelerometers are available, two aluminum blocks are needed.)
Oscilloscope. (If two accelerometers are available, a dual-trace oscilloscope is desirable.)
Oscilloscope camera. (This piece of equipment is not absolutely required, but would greatly simplify the experiment.)
Electric motor and adjustable counterweights as in Figure 11 (can be nuts screwed into threaded rod.)
Guard for counterweights. (See Note before first step below.)

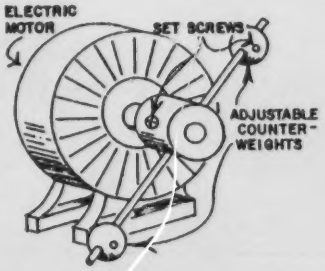


FIG. 11

PROCEDURE

Note: This experiment requires a guard to be placed over rotating weights to avoid possible accident. Have instructor check out system before starting experiment.

1. Set up experiment as shown in Figure 12. Have counterweights balanced and guards in place.

2. Turn on motor; measure acceleration of motor mount and support surface at points A and B. Calculate transmission:
   $T = \frac{a_s}{a_m}$; record in Data Table as Case 1.

3. Turn off motor; move one counterweight either toward or away from motor axis, producing a slight imbalance.

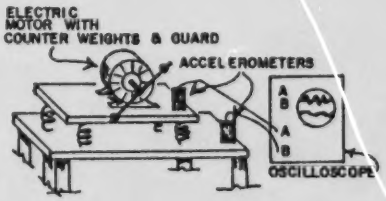


FIG. 12

4. Repeat step 2; record results as Case 2 of Data Table. If resulting vibrations are not large enough, turn off motor and increase imbalance of counterweights.

5. Turn off motor; balance counterweights. Repeat step 2,

recording results as Case 3 of Data Table.

6. Are vibrations produced by the motor-counterweight arrangement used in experiment section external or internal?

DATA TABLE				
	a_m	a_s	$T = \frac{a_s}{a_m}$	Class of Vibration
Case 1				
Case 2				
Case 3				

SELF CHECK

1. Distinguish between vibrations produced by internal and external sources. Give two examples of each type; prepare a labeled sketch of each.

2. In one or two sentences define these terms:

   Steady-state vibration.
   Transient vibration.
   Periodic motion.
   Amplitide.
   Isolation.
   Superposition.
   Frequency.
   Period.

3. A rock crusher has an acceleration of 5.5 cm. It is spring mounted on a wooden floor, which has a measured acceleration of 3 cm. What is the value for transmission of the vibrations?

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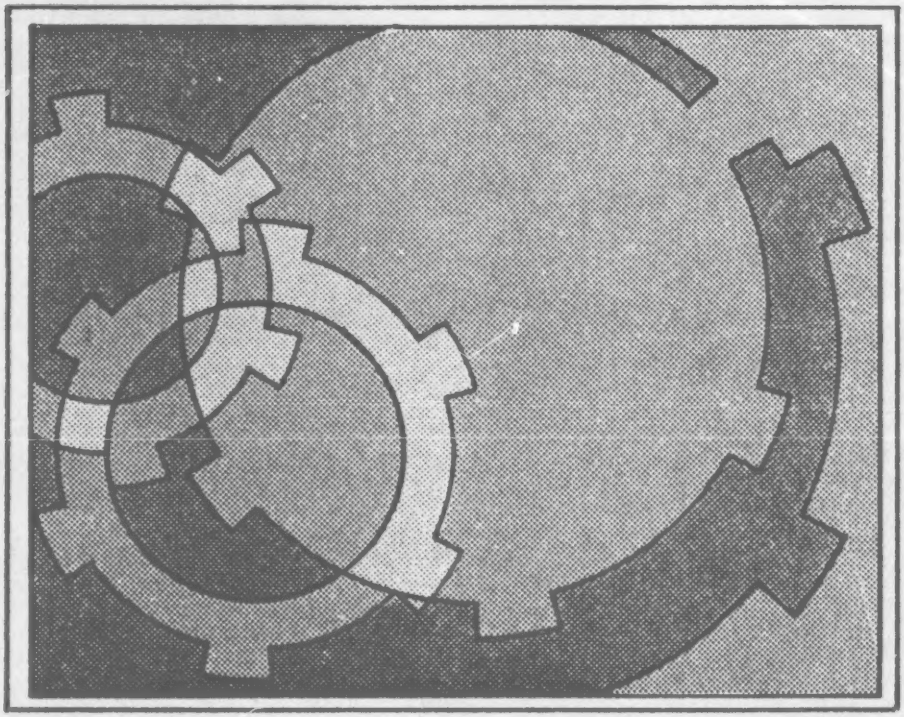
UNIFIED TECHNICAL CONCEPTS

MODULE 11 M 4

RESONANCE



MECHANICAL APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center - Southwest TERC - SW

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INTRODUCTION

Each oscillating energy system has one (or more) frequency at which it exhibits its strongest oscillations, called the resonant or natural frequency or frequencies of the system. Energy is most effectively added to such a system when the frequency of the energy source matches a resonant frequency of the system. This module discusses resonance (frequency matching), and gives examples of conditions in which resonance is either an advantage or a disadvantage. Experiments that involve two mechanical systems are used to demonstrate resonance.

PREREQUISITES

Student should have completed Concept Module II-O, "Vibrations and Waves," Application Module IIE1, "Basic Characteristics of Alternating Voltage and Currents," and Application Module IIF1, "Cavity Vibrations and Standing Waves."

OBJECTIVES

To complete this module, the student should be able to:

- Define:

  Natural frequency.
  Resonant frequency.
  Sympathetic vibration.
  Driving frequency.

- Describe the phenomenon of energy transfer (from a driving to a secondary system) when resonance is achieved.

- Describe two cases in which resonance is an advantage, and two cases in which resonance is a disadvantage.

- Calculate the natural frequency of a pendulum, given its length; observe energy transfer from one pendulum to another when their natural frequencies are equal.

- Calculate the resonance frequencies of a closed column of air, given the speed of sound and the length of the air column; observe transfer of acoustic energy from a tuning fork to an air column.

DISCUSSION

All oscillating energy systems have certain frequencies at which they oscillate easily. These frequencies - the ones for which standing waves may be formed - are called the natural frequencies of the system. Because they also are the frequencies at which resonance occurs, they sometimes are referred to as the resonant frequencies.

Resonance is an energy transfer process in which a large amplitude oscillation is established in a system at one of its natural frequencies by driving it with a small amplitude oscillation at that frequency for a period of time. The applied signal is called the driving frequency; the resulting oscillations are termed sympathetic vibrations.

FIG. 1

Figure 1 shows a simple and familiar example of resonance. The driving force in this example is the push given to the swing after it comes to rest at its maximum height on each return motion. The sympathetic vibrations appear as increased amplitude of the swing-like motion. A single push on the swing will have little effect on this amplitude, but a series of small pushes at the proper time will cause a gradual increase in the maximum amplitude.

The period of a pendulum is the time required for it to complete one cycle of its repetitive motion. In Figure 2 this period is the time for the pendulum to swing from its original position to its maximum height in the opposite direction (dotted) and back to the original position. The period of a pendulum is given in Equation 1.

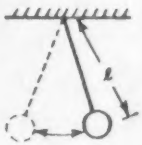

FIG. 2

$$T = 2\pi \sqrt{\ell/g} \qquad (1)$$

where

T = period of pendulum.
ℓ = length of pendulum.
g = acceleration of gravity (9.8 m/s^2).

Example A shows how this equation is used in solving a problem.

EXAMPLE A: PERIOD OF A PENDULUM

Given: Pendulum length: 3 m.

Find: Pendulum period.

Solution:

$$T = 2\pi\sqrt{\ell/g}$$

$$= 2 \times 3.14 \times \sqrt{3m/9.8\ (m/s^2)}$$

$$= 6.28 \times \sqrt{0.306\ s^2}$$

$$= 6.28 \times 0.55\ s$$

$$T = 3.48\ s$$

The swing of this pendulum will increase in amplitude if it is given a push every 3.48 seconds, which is its natural or resonant frequency. Any other driving frequency will not produce the maximum amplitude.

The closed organ pipe shown in Figure 3 is another example of resonance. Air rushing past the baffle produces a hissing sound that contains many frequencies, including resonant frequencies of the closed pipe; these resonant frequencies produce standing acoustic waves in the pipe. The amplitudes of these waves increase to produce the characteristic sounds of the organ pipe.

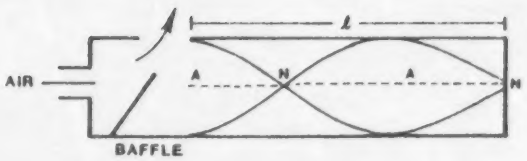

FIG. 3

Other sound frequencies do not produce standing waves in the pipe, and their amplitudes remain too low to be heard. The forward and reflected waves at these frequencies effectively cancel one another out.

The resonant frequencies of the pipe are those that produce a node at the closed end and an antinode at the opening. They are given by Equation 2.

$$f_m = \frac{mv}{4\ell} \qquad (2)$$

where
- m = odd integer (1, 3, 5, ...)
- v = velocity of sound.
- ℓ = length of pipe.

This type of resonant energy system has a large number of resonant frequencies. The lowest of these, corresponding to $m = 1$, is called the "fundamental" frequency. Example B shows the use of Equation 2 in calculating some of these frequencies.

EXAMPLE B: RESONANT FREQUENCIES OF A CLOSED ORGAN PIPE

Given: Closed organ pipe length: 2m; velocity of sound: $v = 346$ m/s.

Find: First three resonant frequencies.

Solution:

$$f_m = \frac{mv}{4\ell}$$

a. $$f_1 = \frac{1 \times (346\ m/s)}{4 \times (2m)}$$

$$= \frac{346\ m/s}{8\ m}$$

$$= 43.25/s$$

$$f_1 = 43.25\ Hz$$

b. $$f_3 = \frac{3 \times (346\ m/s)}{4 \times (2m)}$$

$$f_3 = 129.75\ Hz.$$

c. $$f_5 = \frac{5 \times (346\ m/s)}{4 \times (2m)}$$

$$f_5 = 216.25\ Hz$$

Note that the higher frequencies also can be found by using $f_m = mf_1$.

Resonance, a common phenomen, is used in a variety of applications. The sounds of many musical instruments are produced by

resonance in enclosed volumes of air. The ear responds to these sounds because tiny fibers in the inner ear are caused to vibrate at their resonant frequencies.

Electronic circuits containing inductors and capacitors also have resonant frequencies. Electronic signal generators are devices that cause standing electronic waves to be established at these natural frequencies.

Tuners in radios are LC circuits with variable capacitors. When the circuit is adjusted to resonate at the same frequency as a radio station, the amplitude of the signal in the circuit increases to a point at which it may be amplified and converted to sound. Stations on other frequencies do not produce resonance and are not heard.

There are such cases in which resonance must be avoided. If a structure is subjected to a vibration at one of its resonant frequencies, damaging oscillations may result. The most striking example of this was the collapse of the Tacoma Narrows Bridge. Wind exerted a pulsating force oh the bridge at one of its natural frequencies. This resonant condition allowed transfer of energy from the wind to the bridge and the amplitude of the vibrations increased, which lead to the collapse of the structure after only a few hours. Since that incident, suspension bridges have been designed to have resonant frequencies far different from the frequencies of natural winds.

Another example of unwanted resonance is the oscillation produced when the wheels of an automobile are not properly balanced. The suspension system of the automobile has a natural frequency at which it will oscillate. At a particular speed the frequency of the vibrations produced by an unbalanced wheel are equal to the resonant frequency of the suspension system; the resulting sympathetic vibrations cause a bumpy ride. The same phenomena can occur in shock mounts for machinery if the vibration frequency of the machinery is near the resonance frequency of the support structure.

Other common examples of resonance include dishes or windows that vibrate when a train or a large truck passes. The vehicle produces sound containing many frequencies. Sympathetic vibrations occur in objects at their natural frequencies if those frequencies are present in the incident sound waves.

SUMMARY

Many mechanical, electrical and fluid systems have natural frequencies at which they oscillate easily. Resonance occurs whenever pulsating energy is delivered to these systems at their natural (resonant) frequency. Some of the incident

energy is converted to a wave motion in the system called a sympathetic vibration. In some cases, such as radio tuners and clocks, these resonant frequencies are desirable and useful. In other, such as mechanical structures and d.c. power supplies, they are troublesome and must be eliminated.

RESONANCE OF PENDULUMS

MATERIALS

String (approximately 5m).
Four 100-gram weights.

PROCEDURE

1. Tie length of string between two vertical supports, as shown in Figure 4.
2. Tie a piece of string to each weight and to horizontal string to form four pendulums with lengths indicated in Figure 4.
3. Start right pendulum of Figure 4 swinging in plane perpendicular to horizontal string; observe motion of all four pendulums for several minutes and describe in Data Section.
4. Record length of each pendulum in Data Section; calculate period of each (use Equation 1).
5. Explain in a short paragraph how this experiment demonstrates resonance.

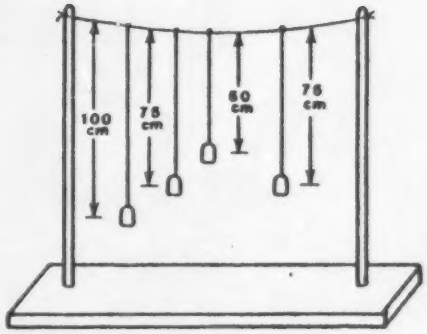

FIG. 4

RESONANCE IN ORGAN PIPES

MATERIALS

Resonance tube (Sargent-Welch #3308) consisting of the following components:

Glass tube 1-inch in diameter and 4-feet long.
One hole stopper for tube.

Flexible tubing.
Container for water.
Meter stick.
Mounting brackets.
Centigrade thermometer.
Three tuning forks with the following frequencies:
128 Hz.
256 Hz.
512 Hz.

PROCEDURE

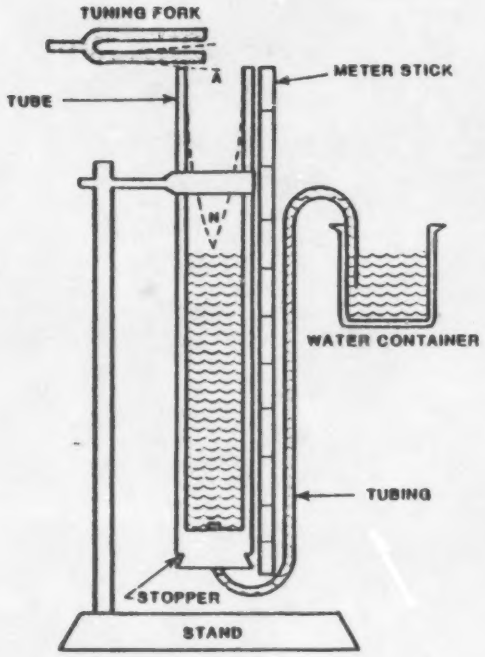

FIG. 5

1. Measure air temperature; record in Data Section.
2. Calculate velocity of sound in air.
3. Calculate length of tube required for resonance at frequency of each tuning fork (use Equation 2); record lengths in Data Section.
4. Set up equipment as shown in Figure 5.
5. Fill glass tube and flexible tube with water; adjust water level in glass tube by raising or lowering water container until length of tube containing air is 2 or 3 cm less than calculated length for resonance for lowest frequency tuning fork.
6. Strike lowest frequency fork and hold over tube end, as shown in Figure 5; simultaneously, lower water container to allow water to drain from tube; listen to tone produced. (Resonance, which will occur when correct air length is reached, is indicated by maximum amplitude of sound.) Repeat this step with water level rising through resonance point; record average resonant length in Data Section.
7. Repeat steps 5 and 6 for other two tuning forks.
8. In a brief paragraph, explain what occurs in the air column at resonance.

RESONANCE OF PENDULUMS

Describe the motions of each of the four pendulums during the experiment performed in the Laboratory procedures:

Periods of Pendulums:

Pendulum	Length (m)	Period (s)
1		
2		
3		
4		

RESONANCE IN ORGAN PIPES

Air Temperature: T = _____°C

Velocity of Sound in Air: V = 331 m/s + (0.6 m/s - C°) x t

= 331 m/s + (0.6 m/s - C°) x (____C°)

= _____ m/s.

Frequency of tuning fork (Hz)	Calculated Length of air column for first resonance point (m) $\ell = \frac{v}{4f}$	Measured Length of air column for resonance (m)

SELF CHECK

1. Identify the terms that match the following definitions:

   a. The frequency at which an oscillating energy system vibrates most easily (two names).
   b. The vibrations occuring in a system when it receives energy at one of the above frequencies.
   c. The frequency of the energy applied to a resonant system.

2. Describe how energy is transferred to a pendulum when a force is applied at its resonance frequency.

3. Describe how resonance is used advantageously in a radio tuner.

4. Describe how resonance can be a disadvantage in structures (such as bridges).

5. Calculate the period of pendulums with each of the following lengths:

   a. 25 cm.
   b. 5 m.

6. Calculate the fundamental frequency of a closed organ pipe with each of the following lengths:

   a. 50 cm.
   b. 37.5 cm.

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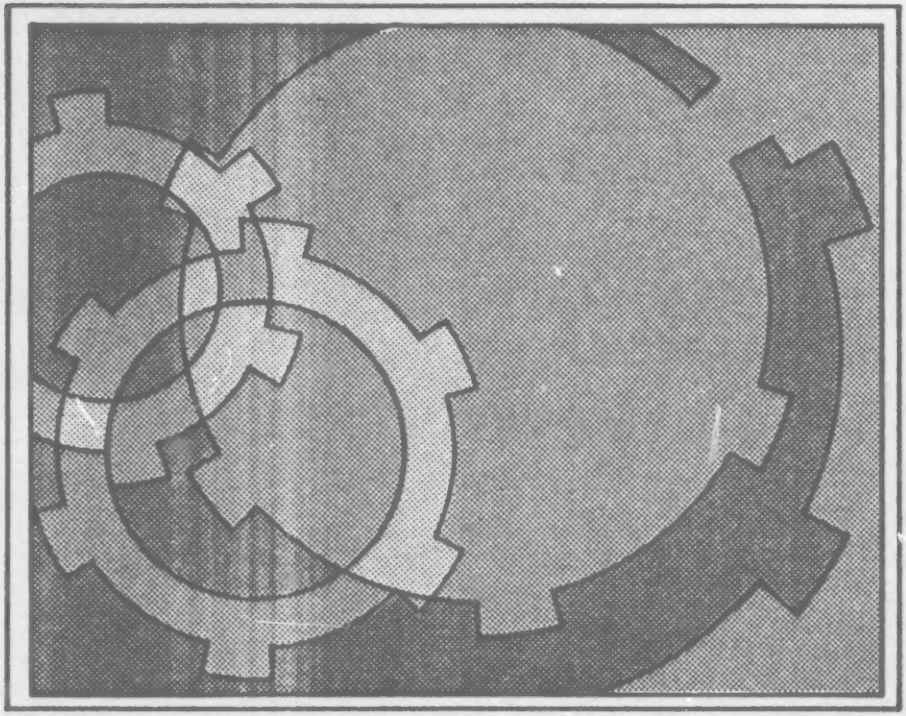
UNIFIED TECHNICAL CONCEPTS

MODULE 11 M 5

SOUND SPEAKERS



MECHANICAL APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

A sound speaker is a device which converts electrical energy into sound energy. In this module the design and performance of several types of speakers will be examined. Emphasis will be placed on construction details of speakers to produce high fidelity sound over specific frequency ranges. In the laboratory the student will measure the frequency response of a moving coil speaker.

PREREQUISITE

Student should have completed Concept Module II-0, "Vibrations and Waves," Application Module IE4, "Magnetic Forces and Fields."

OBJECTIVES

To complete this module, the student should be able to:

- Diagram and label the primary components of a moving coil speaker.
- Describe in a short paragraph how the moving coil speaker produces sound waves.
- In two or three sentences, describe the design characteristics of each of the following types of speakers.

  Woofer (low frequency).
  Tweeter (high frequency).
  Mid-range.
  Wide-range.

- Diagram and label the three primary components of an electrostatic speaker.
- Describe in a short paragraph how an electrostatic speaker produces sound waves.

- Given the appropriate equipment, measure the frequency response of a speaker.

DISCUSSION

The sound speaker is one of the most common elements of modern communication systems. Figure 1 shows a cross-section diagram of a moving coil speaker. Sound waves are produced by motion of the speaker dome and cone. These structures must be both rigid and light-weight, and are commonly made of stiff paper. The inner and outer suspensions allow the cone and dome to vibrate and typically consist of flexible folds of paper or cloth. The voice coil of the speaker is rigidly attached to the cone and dome. Flexible leads connect the moving coil to terminals located on the cone basket. The lower portion of the speaker contains a permanent magnet and a yoke, constructed to give a uniform mangetic field at the voice coil.

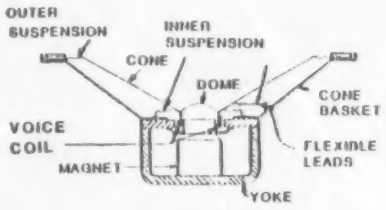


Fig. 1

Figure 2 shows the voice coil and the magnetic field produced by the magnet and yoke. An electron current flowing through the conductor of the coil crosses the magnetic field lines producing an upward force on the voice coil. Reversing the direction of the electron current produces a downward force on the voice coil. Since the coil is rigidly attached to the cone and dome, these components are driven up and down by an alternating current (a.c.) signal applied to the coil. This motion of the cone and dome produces the compression and rarefaction of sound waves.

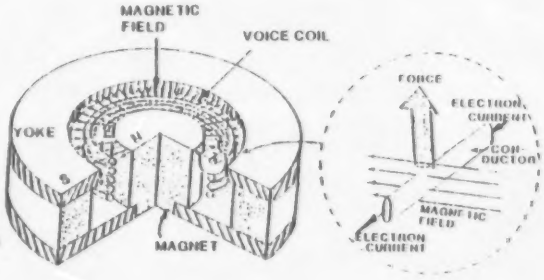


Fig. 2

A primary concern in designing or selecting a speaker is the frequency range over which it is useful. The human auditory range extends from about 20 Hz to 20,000 Hz. High fidelity sound reproduction requires speakers that can produce any frequency within this range. The following are characteristics of moving coil speakers that limit their frequency response:

* Good low frequency performance requires a large area cone that can move a relatively large volume of air. The voice coil and magnet of such a speaker must be large enough to provide the necessary force to move the cone and the air.

* Good high frequency performance requires that the mass of the moving components be as small as possible and that the cone and dome be as rigid as possible.

Clearly a single speaker cannot meet both of the above requirements. The design characteristics of speakers for three different frequency ranges will now be examined.

"Woofers" are large speakers with excellent low frequency response. Since the best low frequency output is produced by the largest cone area, "woofers" over one foot in diameter are common and 18 inch diameter speakers are not rare. "Woofers" commonly produce sound from about 20 Hz to 2,000 Hz.

"Tweeters" are small speakers for the production of high frequency sound. The voice coil, cone and dome of a tweeter must be as light as possible. Since moving large volume of air is unnecessary and the sound producing components must be very rigid, the cone of a tweeter is small. In many cases it is eliminated completely and sound is produced by the vibrating dome alone. Tweeters are commonly two or three inches in diameter and produce sound in the range of 5000 Hz to 20,000 Hz.

Mid-range speakers are a compromise between the design characteristics of the woofer and the tweeter. They are 4 or 5 inches in diameter and perform well in the range of 2000 Hz to 10,000 Hz.

The typical response curves of the three types of speakers described above are shown in Figure 3. High fidelity sound systems employ all three types to cover the entire range of human hearing. The driving signal for such a speaker system is split into separate parts going to each type of speaker by passing through an electronic filter circuit that allows only certain frequency ranges to pass. Thus, the woofer is driven by the low frequency components, the mid-range speaker receives the mid-frequencies of the signal and the high frequency components go only to the tweeter.

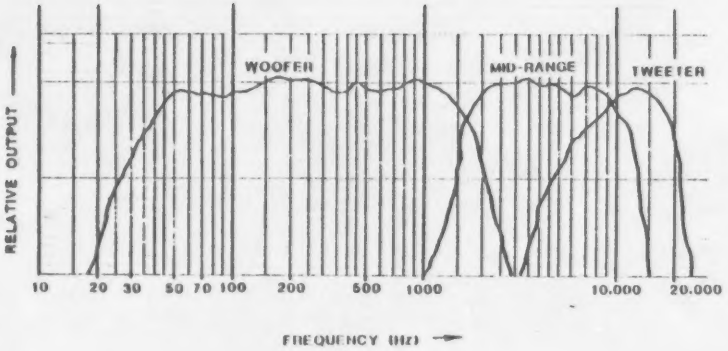


Fig. 3

In sound systems such as portable radios, tape players and automobile stereos, it is desirable to produce the best possible sound with a single speaker. This is accomplished with the wide range speaker shown in Figure 4. The cone of this speaker is divided into three zones (1, 2 and 3). The inner cone (1) is a highly rigid structure designed for good high frequency performance. The outer cone (2) is larger and more flexible, providing good low frequency performance. The transistion region is more flexible still, transmitting low frequency vibrations to the outer cone while allowing almost independent high frequency vibrations of the inner cone.

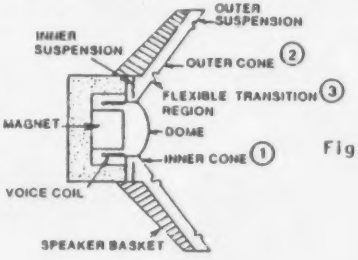


Fig. 4

Figure 5 shows the relative response of this type of speaker. Notice that its output drops off at both high and low frequencies.

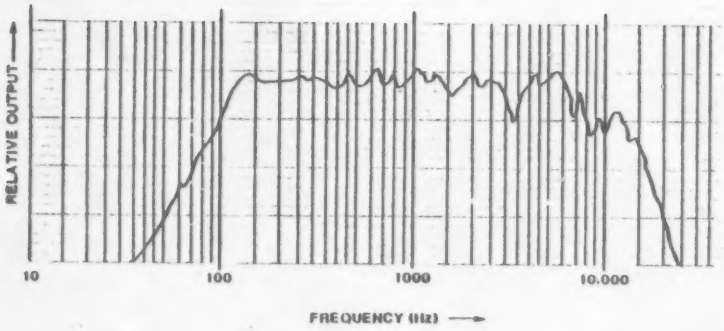


Fig. 5

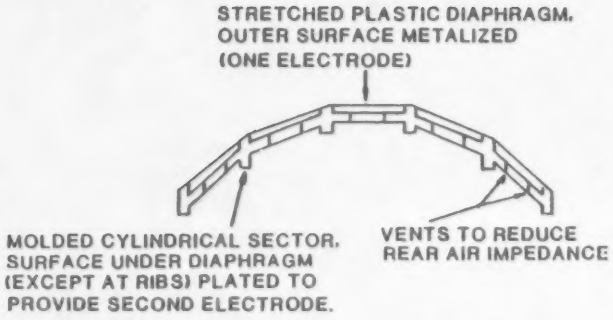

STRETCHED PLASTIC DIAPHRAGM. OUTER SURFACE METALIZED (ONE ELECTRODE)
MOLDED CYLINDRICAL SECTOR. SURFACE UNDER DIAPHRAGM (EXCEPT AT RIBS) PLATED TO PROVIDE SECOND ELECTRODE.
VENTS TO REDUCE REAR AIR IMPEDANCE

Fig. 6

Figure 6 shows a different type of speaker construction that is sometimes used for "tweeters." It is called an electrostatic speaker and employs an electric field instead of the magnetic field of a moving coil speaker. The electrostatic speaker is essentially a capacitor with one movable electrode. The fixed electrode is a molded cylindrical form with ribs to support a stretched plastic diaphragm. The outer surface of the diaphragm and the upper surface of the cylinder are metalized to form the capacitor plates. Vents in the cylindrical surface allow air to flow to and from the small enclosed space to reduce resistance due to pressure differences. Changes in the applied electric field cause the plastic diaphragm to vibrate, thereby producing sound waves. Because of the extremely low mass which must be moved, electrostatic speakers have excellent high frequency response.

MATERIALS

Audio signal generator.
Oscilloscope.
Speaker (an inexpensive 4 inch speaker is recommended but any speaker may be used.)
Microphone.
Clip leads.
Audio amplifier.

PROCEDURE

1. Connect equipment as shown in Figure 7. Microphone should be located directly in front of the speaker at a distance of 4 to 6 inches.

2. Turn on audio signal generator; set frequency selector to about 2000 Hz; set output amplitude to a convenient level.

3. Turn on oscilloscope; set time base to 0.5 ms/cm; set voltage sensitivity to 1 V/cm.

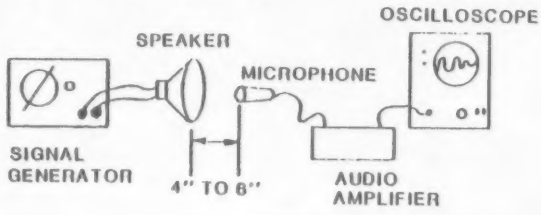


Fig. 7

4. Turn on audio amplifier; adjust gain of audio amplifier and voltage sensitivity of oscilloscope to obtain a signal with a height of about half the oscilloscope scale.

5. Slowly scan frequency selector of signal generator through frequency range of 200 Hz to 20,000 Hz while watching the displayed trace on the oscilloscope. Locate the frequency which produces the greates amplitude signal and set signal generator to that frequency.

6. With signal generator set as in step 5 above, adjust gain of audio amplifier to produce a displayed signal with a height equal to the maximum height of the scale on oscilloscope face. All amplitude settings must remain unchanged for the remainder of experiment.

7. Measure and record height (amplitude) of signal on the oscilloscope for signal generated frequencies ranging from 200 Hz to 20,000 Hz. Take amplitude data at each frequency listed on horizontal scale of the graph (see Figure 8) in the data table. Also take data at any maxima and minima encountered.

8. Divide value of each amplitude by maximum amplitude to obtain relative output.

9. Plot data on the graph paper (Figure 7) provided in data table.

10. Evaluate Figure 7 after the plot is completed and make a short summary statement concerning the speaker response between 200 Hz and 20,000 Hz.

DATA TABLE		
Frequency (Hz)	Amplitude (cm)	Relative Output

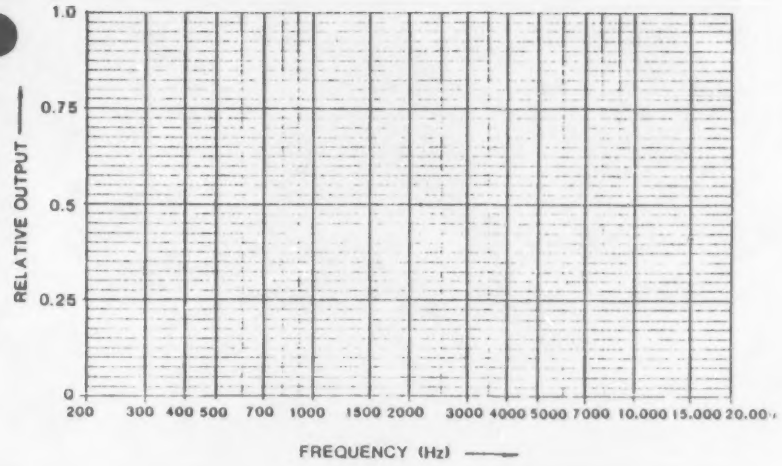

1.0
0.75
0.5
0.25
0
RELATIVE OUTPUT
200
300
400
500
700
1000
1500
2000
3000
4000
5000
7000
10.000
15.000
20.000
FREQUENCY (Hz)

Fig. 7

SELF CHECK

1. Diagram and label the primary components of a moving coil speaker.

2. Describe how the moving coil speaker produces sound waves.

3. Describe the design characteristics of each of the following types of speakers:

   Woofer (low frequency)
   Tweeter (high frequency)
   Mid-range
   Wide-range

4. Diagram and label the three primary components of an electrostatic speaker.

5. Describe how an electrostatic speaker produces sound waves.

REFERENCES

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McGraw-Hill Encyclopedia of Science and Technology, Vol. 7. New York: McGraw-Hill Book Company, 1960.

Neighbor, J. Edward and Newbower, Ronald S. "The Loudspeaker, A Module on Sound." Cambridge, MA: Technical Education Research Center, 1973.

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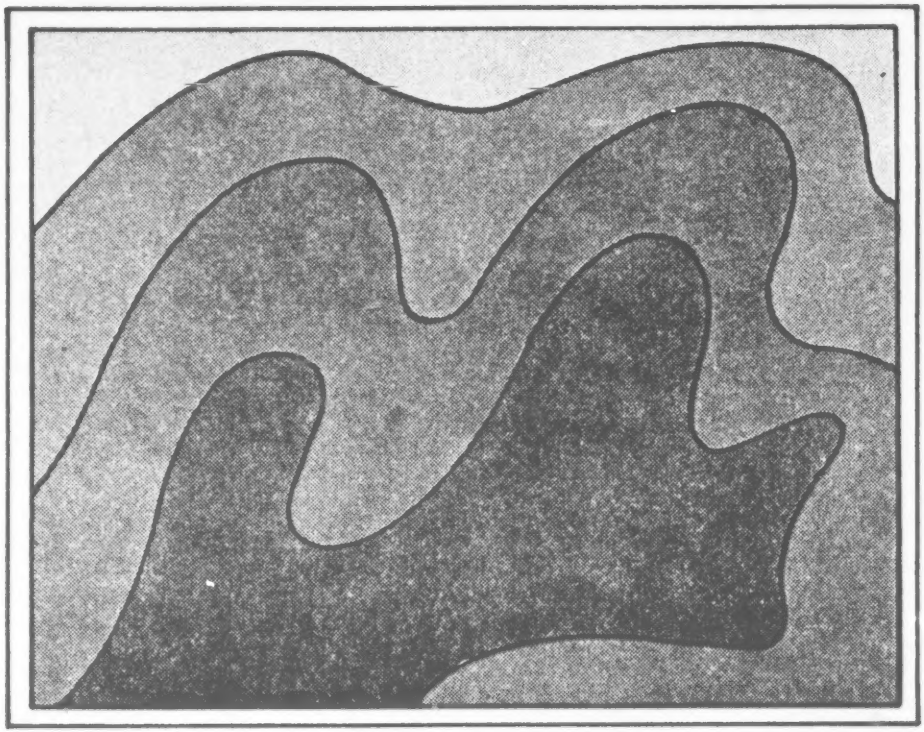
UNIFIED TECHNICAL CONCEPTS

MODULE 11 F 1

CAVITY VIBRATIONS AND STANDING WAVES



FLUID APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

Many devices require, to some extent, the formation of standing waves to be able to perform their functions. These devices include musical instruments, microwave generators and lasers. Standing waves are produced when two like waves of equal amplitude and frequency interfere with each other while traveling in opposite directions. A cavity is a volume containing a medium that supports wave reflection at the ends of the volume. This module investigates the nature of standing waves and conditions necessary for their formation. In the laboratory, the student will produce acoustic standing waves in an "organ pipe" and determine characteristics of cavity vibrations.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Given any two of the following quantities, calculate the third:

  Wavelength.
  Frequency.
  Wave velocity.

- Define the term "standing wave."

- Given the wave velocity and length of a resonant cavity, calculate resonant frequencies.

- Given appropriate equipment, construct a resonant acoustic cavity and determine its resonant frequencies.

DISCUSSION

A standing wave is formed when two like waves of equal amplitude and frequency interfere with one another while traveling in opposite directions. The waves may be transverse waves, such as waves in a string, or longitudinal waves, such as sound waves. The resultant is a wave that appears to "stand still" in space. That is, certain regions exist wherein displacement is quite large, while other regions exist wherein there is little or no displacement. Figure 1 illustrates how this occurs and why the wave appears to stand still.

In this figure, two waves (dotted and solid lines) of equal amplitude and frequency travel in opposite directions. The waves are exactly out of phase at t_0, which produces a resultant represented by a straight line. At t_1 (1/4 the wave period later), the solid wave has moved 1/4 a wavelength to the right, and the dotted wave has moved 1/4 a wavelength to the left. This movement places the two waves in phase and gives the maximum resultant shown. At t_2 the waves are out of phase again. At t_3 they are in phase again to produce a maximum displacement. At t_4 the period of each wave is completed and the process repeats.

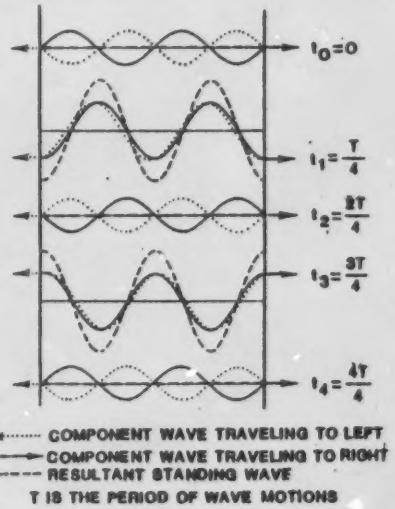


FIG. 1

As time proceeds, the standing wave shows points of maximum vibration (antinodes) separated by points of zero vibration (nodes) (A and N in Figure 2). At antinodal positions (A), the resultant wave moves up and down between maximum displacement limits shown (arrows in Figure 2). At nodal positions (N), no up and down motion occurs. Overall, there is no apparent motion of the resultant wave in a left-right direction. Hence, the resultant wave appears to stand still even though individual waves are moving to the left and right.

Figure 3 shows several possible standing waves in a string tied at both ends. Since the string cannot vibrate at the ends, there must always be a node there. Wave velocity remains constant, depending only on the tension in the string and its density. Wavelength and frequency of each pattern or mode are given in the figure. Notice that relationship between wavelength and string length can be found by inspection of the figure. Once the wavelength for any mode is determined, corresponding frequency

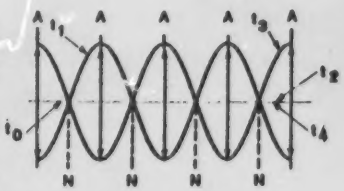


FIG. 2

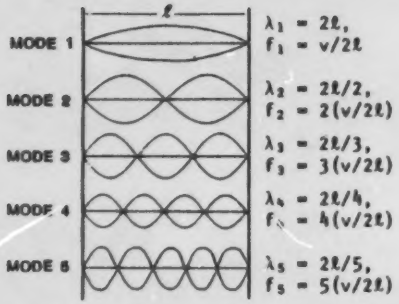


FIG. 3

can be found from Equation 1.

$$f = \frac{v}{\lambda} \qquad (1)$$

where
f = frequency.
v = wave velocity.
λ = wavelength.

Notice that mode spacing - the frequency difference between two adjacent modes - is a constant given by Equation 2.

$$\Delta f = \frac{v}{2\ell} \qquad (2)$$

where
Δf = mode spacing.
ℓ = cavity length.

Frequency of any mode can be determined by multiplying this frequency by the mode number, as given in Equation 3:

$$f_n = n\Delta f \qquad (3)$$

where
f_n = frequency of nth mode.
n = mode number.

Example A shows use of Equations 1-3 in solving a problem.

Resonant frequencies of a system are those frequencies for which a standing wave can exist. In Example A, any multiple of 500 Hz is a resonant frequency. No other frequency can form a standing wave since it would not produce a node at both ends of the string.

Figure 4 shows the first four modes possible in an organ pipe with both ends open. In this case, the air executing longitudinal vibrations is free to move at each end. This results in an antinode at each end of the pipe. Equations developed from Figure 3 hold for this example also.

A "cavity" is any volume containing a medium that supports wave propagation in such a manner that some reflection of the waves occurs at the ends of the volume. In the case of the vibrating string, reiflections occur at the tied ends. In the case of the

EXAMPLE A: CALCULATION OF MODE SPACING, FREQUENCY & WAVELENGTH

Given: From Figure 3, $\ell = 1$ m and $v = 1000$ m/s.

Find: (a) mode spacing, (b) frequency of third mode and (c) wavelength of third mode.

Solution: (a) $\Delta f = \frac{v}{2\ell}$

$$= \frac{1000 \text{ m/s}}{2 \times (1 \text{ m})}$$

$$\Delta f = 500 \text{ Hz}$$

(b) $f_n = n\Delta f$

$$f_3 = 3 \times 500 \text{ Hz}$$

$$= 1500 \text{ Hz}$$

(c) $\lambda = \frac{v}{f}$

$$= \frac{1000 \text{ m/s}}{1500 \text{ Hz}}$$

$$= 0.67 \text{ m}$$

Notice that in Figure 3, wavelength of the third mode is 2/3 ℓ, or 0.67 m, as determined in this example.

organ pipes, air transmits sound waves, and open ends of the pipes actually produce partial reflection of the waves. In the optical cavity of a laser, light is reflected by a mirror at each end. In each case, reflection of waves at the ends of the cavity provides a pair of identical waves traveling in opposite directions. If frequency of these waves is a resonant frequency of the cavity, a standing wave is formed.

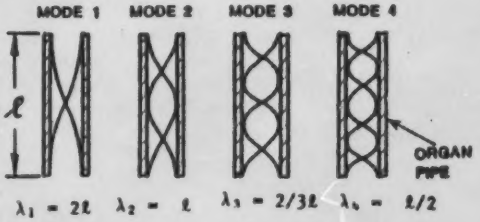


FIG. 4

Resonant cavities are employed when a pure frequency is to be produced. Most musical instruments use resonant cavities consisting of columns of air (organ pipes, trumpets) or stretched strings (pianos, violins). Klystron tubes use a metallic-walled cavity to produce standing waves in the microwave region of the electromagnetic spectrum. Such tubes are employed in radar and communication.

Figure 5 shows elements of a klystron tube. Electrons from the cathode are accelerated toward the anode. They pass through the grids into the drift space. They are reflected back through the upper grid by the repeller. Velcoity changes of the electrons passing through the cavity produce microwave radiation. The cavity of the tube is a resonant cavity for the wavelength of micro-

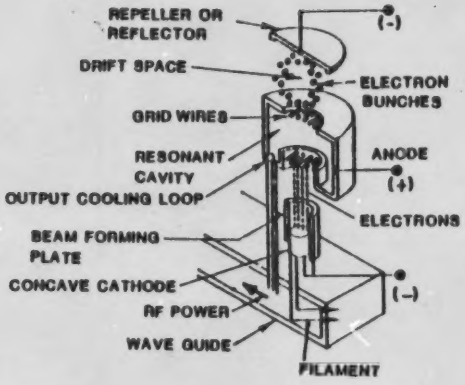


FIG. 5

waves to be produced, and standing waves form within it. Part of the energy is transmitted out of the tube by the output coupling loop. Wavelength of the microwaves produced can be altered by changing the shape of the resonant cavity.

SUMMARY

Standing waves result from the interference (constructive or destructive) produced by the contact of waves traveling in opposite directions. Many lasers, microwave generators, musical instruments and other devices require some formation of standing waves. Waves in a cavity reflect at the ends of the cavity to produce a pair of identical waves traveling in opposite directions; if these wave pairs are in resonance, they are standing waves.

STANDING WAVES AND CHARACTERISTICS OF CAVITY VIBRATIONS

MATERIALS

Oscilloscope.
Audio frequency signal generator.
Speaker.
Microphone.
Cardboard or plastic tube approximately 3 inches in diameter and 4 feet long, open at both ends.
Similar tube approximately 2 feet long.
Clip leads.

PROCEDURES

1. Measure length of each tube; record in Data Table.

2. Set up equipment as shown in Figure 6, using longer of two tubes. Set oscilloscope for 0.005 v/cm and 2 ms/cm. Turn on signal generator; set frequency control at approximately 200 Hz.

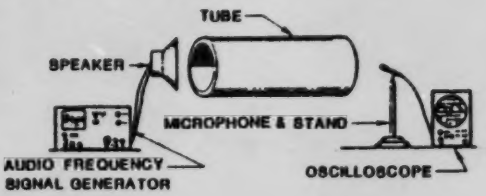


FIG. 6

3. Slowly increase frequency produced by signal generator while watching display on oscilloscope. Amplitude of displayed audio signal will increase and decrease as frequency increases. Maximum amplitude occurs when frequency of sound waves is a resonant frequency of open tube and acoustic standing waves are formed in tube. Measure at least four of these frequencies with oscilloscope; record them in order in Data Table [lowest frequency (f_1) first, and so on up to f_4].

4. Complete calculations in part A of Data Table.

5. Complete predicted results portion of part B of Data Table, using velocity of sound in air determinud in part A.

6. Replace original tube with shorter one; determine its resonant frequencies. Record these in experimental-results

section of Data Table.

7. Compare predicted and experimental resonant frequencies in part B of Data Table.

8. How does the cavity size formed by the tubes effect wave resonance?

DATA TABLE

PART A

Length of first tube: ℓ = _____ m

Resonant frequencies: f_1 = _____ Hz
f_2 = _____ Hz
f_3 = _____ Hz
f_4 = _____ Hz

Mode spacings: $f_4 - f_3$ = _____ Hz
$f_3 - f_2$ = _____ Hz
$f_2 - f_1$ = _____ Hz

Average modue spacing:

$$\Delta f = \frac{1}{3}[(f_4 - f_3) + (f_3 - f_2) + (f_2 - f_1)]$$

$$= \frac{1}{3}[\_\_\_\_\_ \text{Hz} + \_\_\_\_\_ \text{Hz} + \_\_\_\_\_ \text{Hz}]$$

Δf = _____ Hz

Since $\Delta f = \frac{v}{2\ell}$,

velocity of sound: $v = \Delta f \times 2\ell$
= _____ Hz x 2 x _____ m
v = _____ m/s

PART B

Length of second tube = _____ m

Predicted results:

Mode spacing: $\Delta f = \frac{v}{2\ell}$
= _____ m/s ÷ (2 x _____ m)
Δf = _____ Hz

Resonant frequencies: $f_1 = 1 \times \Delta f$ = _____ Hz
$f_2 = 2 \times \Delta f$ = _____ Hz
$f_3 = 3 \times \Delta f$ = _____ Hz
$f_4 = 4 \times \Delta f$ = _____ Hz

Experimental Results:

Measured resonant frequencies: f_1 = _____ Hz
f_2 = _____ Hz
f_3 = _____ Hz
f_4 = _____ Hz

SELF CHECK

1. Velocity of light in vacuum is 3×10^8 m/s. What is the frequency of a light wave with a wavelength of 5×10^{-5} cm?

2. Velocity of sound in air at sea level is 335 m/s. What is wavelength of a sound wave with a frequency of 800 Hz?

3. Define the term "standing wave."

4. An organ pipe (open at both ends) has a length of 3.2 m. What are frequencies of the first three modes of vibration?

5. A laser cavity with mirrors at opposite ends has a length of 25 cm. Calculate mode spacing at the cavity. Use 3×10^8 m/s as velocity of light in the cavity.

6. Predict the first few modes of vibration for an organ pipe closed at one end and open at the other. Give mode frequencies in terms of pipe length ℓ and velocity of sound v in air.

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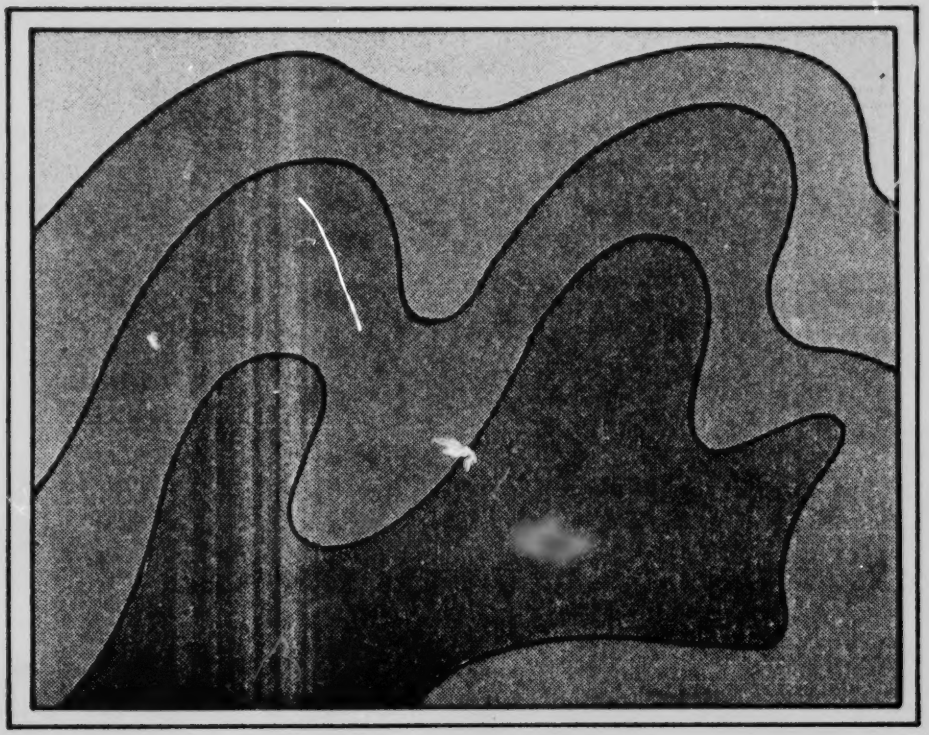
UNIFIED TECHNICAL CONCEPTS

MODULE 11 F 2

SPEAKER ENCLOSURES



FLUID APPLICATION OF

VIBRATIONS AND WAVES CONCEPT

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

In Application Module IIM5, "Sound Speakers," the characteristics of speakers used to produce sound waves were examined. These characteristics alone do not ensure the fidelity of the sound output of a reproduction system. Speakers must be mounted in some way and are usually enclosed in a cabinet or housing. The design of speaker enclosures greatly affects speaker performance. In this module, the characteristics of three types of speaker enclosures will be examined. In the laboratory, the student will measure directionality of sound produced by a speaker with and without an enclosure.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves," Application Modules IIM5, "Sound Speakers," and IIE3, "Oscilloscopes."

OBJECTIVES

To complete this module, the student should be able to:

- Sketch a typical sound radiation pattern produced by a moving-coil speaker.
- Describe in two or three sentences how interference of reflected sound waves affects speaker performance.
- Describe in two or three sentences how horns control the directionality of speakers.
- Describe in a short paragraph how full enclosures improve speaker performance. Include the importance of acoustic absorption within the enclosure and the major disadvantage of full enclosures.
- Describe in a short paragraph how reflex enclosures improve speaker performance at low frequency.
- Given the necessary equipment, measure directionality of the sound produced by a speaker with and without an enclosure.

DISCUSSION

Speakers do not radiate sound equally in all directions. Figure 1 shows directionality of sound produced by a typical moving coil speaker. This speaker radiates sound in forward and backward directions with almost equal intensity, but very little sound is directed to the side of the speaker. Sound is emitted most intensely along the line of motion of the speaker cone.

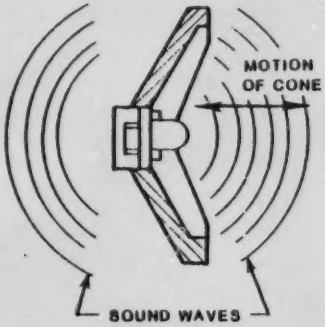


FIG. 1

One way to describe directionality of a speaker is to draw a line connecting all points at which the intensity of sound produced is the same. This line, which is called an "isointensity plot" for the speaker, is produced by measuring sound intensities around the speaker and locating distances from the speaker at which the sound has a specific intensity.

Figure 2 shows isointensity plots for a speaker at three frequencies. In each case, the speaker was located at the center of the pattern with the open end of the cone (speaker axis) directed as shown. At low frequencies, the radiation pattern is almost symmetric about a line perpendicular to the speaker. At higher frequencies, the shape of the pattern is less regular, but the maximum intensity still falls along the speaker axis.

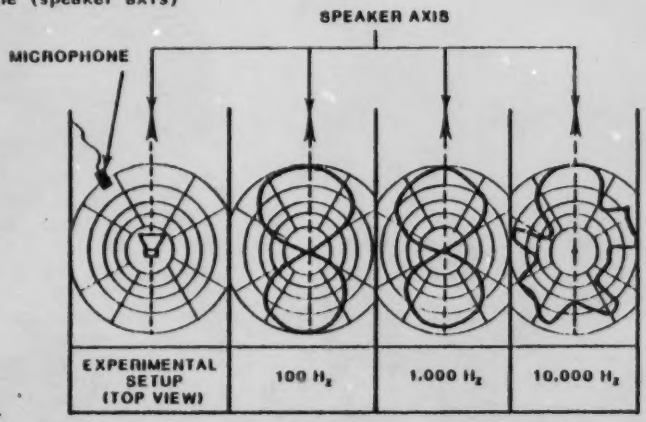


FIG. 2

If a speaker is mounted with a sound-reflecting surface near the rear of the speaker, as shown in Figure 3, a portion of the sound emitted from the rear of the speaker is reflected. For some sound frequencies, illustrated by the lower waves in Figure 3, the reflected wave and the direct wave from the speaker are out of phase, resulting in destructive interference. For other frequencies, illustrated by the upper waves in Figure 3, the two waves are in phase to produce constructive interference. The interference effects produced depend upon both the frequency of sound and the position of the listener. Thus the quality of the sound produced

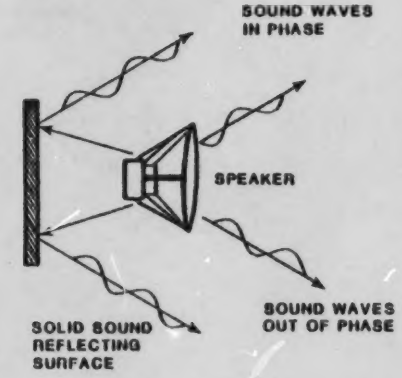


FIG. 3

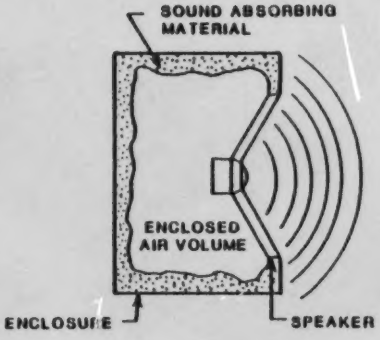


FIG. 4

by such a system may not be consistent as the listener moves through the listening area.

In practical applications of sound reproduction systems, the listener is usually located forward of the speaker. Thus, high-fidelity speaker enclosures are designed to provide the best sound quality in the forward direction. To accomplish this directionality, the speaker must be controlled without introducing unwanted interference effects. The simplest way to do this is to use a full enclosure (Figure 4).

In a full enclosure, all backward-directed sound waves are contained within a closed volume, which eliminates interference in front of the speaker. The enclosure interior is lined with a sound-absorbing material to prevent formation of standing acoustic waves within the enclosed air volume. Failure to eliminate such standing waves results in peaks in the speaker response curve at the frequencies of the standing waves.

A full enclosure is effective at high and mid frequencies of human hearing, but problems arise at lower frequencies. Application Module IIM5, "Sound Speakers," showed that good low-frequency responses can be achieved with large cone areas that can move relatively large volumes of air. Actual displacement of the cone in a "woofer" also is much greater than that of a "tweeter" or a mid-range speaker. The enclosed volume of air in the full enclosure offers considerable resistance to large speaker cones at low frequencies. Low-frequency sound also is difficult to eliminate with sound-absorbing material. For these reasons, full enclosures for low-frequency speakers must be much larger than the speaker dimensions, and they should be well insulated.

In high-fidelity sound systems, one of the most critical problems is to ensure good low-frequency response in the range from 20 Hz to 100 Hz. Because full enclosures always impair speaker performance in this range, to some degree, the bass reflect enclosure shown in Figure 5 is often used. Design details of such enclosures vary greatly. Some contain sound-absorbing materials in a portion of the enclosure. Baffles also may be employed to shape the air cavity inside a bass reflex enclosure; however, the principle is the same: all bass reflex enclosures have 1) internal air volumes that can support standing waves at some low frequencies and 2) openings to allow part of the energy of the standing wave to escape in the same direction as the direct wave from the speaker.

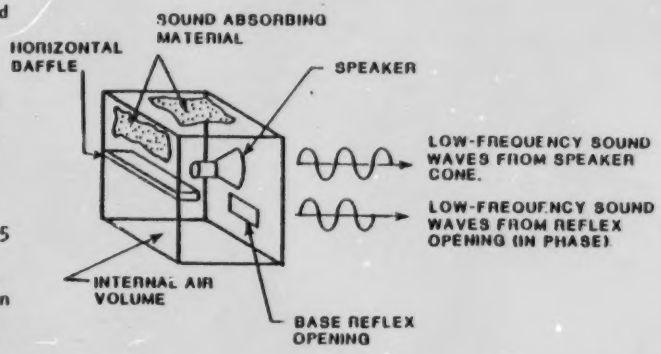


FIG. 5

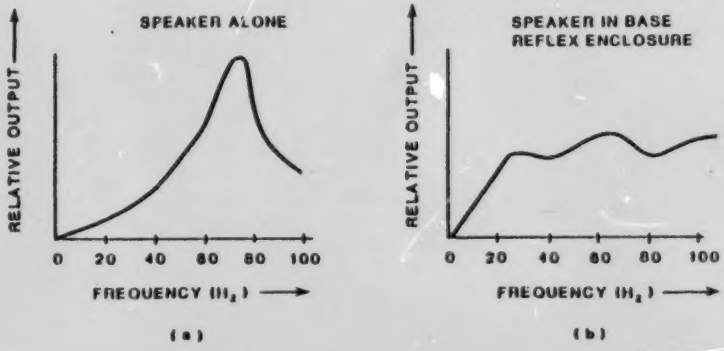


FIG. 6

Design details of such an enclosure are beyond the scope of this module, but it turns out that proper shaping of internal volume as well as choice of area of the reflex opening can greatly alter the low-frequency response of a speaker.

To better understand the overall improvement of a bass reflex system, refer to Figure 6. Graph (a) shows the low-fre-

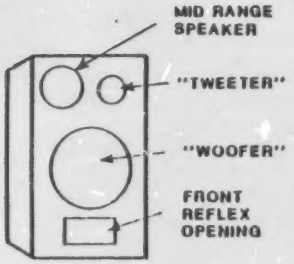


FIG. 7

quency response of a woofer with no enclosure. It is common for such a speaker to have a high peak in the response curve at a resonant frequency of the speaker cone. The best low-frequency speakers are designed to exhibit this peak at frequencies under 50 Hz.

Graph (b) shows the response of the same speaker in a base reflex enclosure. This enclosure has been designed to emit sound (from the reflex opening at 20 Hz) that is in phase with the direct wave from the speaker. The constructive interference of the two waves enhances system performance at this frequency. On the other hand, at the speaker response peak around 70 Hz, sound from the reflex opening is out of phase with the direct wave, and destructive interference lowers the peak. The overall effect is to smooth speaker response at low frequencies, and to extend the useful range of the speaker system to the lower limit of human hearing.

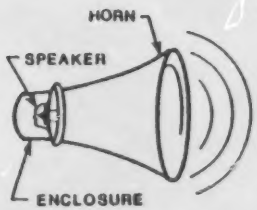


FIG. 8

Figure 7 shows a speaker system for production of high-fidelity sound. Such a system will include a high-frequency speaker, a mid-range speaker and a low-frequency speaker with a reflex enclosure for improved bass performance. The enclosure is of wood or fiberglass, and all joints have been glued or bonded to prevent vibrations. The best enclosures have internal glued braces and blocks at the corners to increase rigidity of the structure. The front of the speaker enclosure is commonly covered with a porous cloth or plastic foam to provide some protection for the speakers while absorbing little of the sound produced.

Figure 8 shows a horn. This sound-controlling device is used primarily in public address systems that require high-intensity sound with controlled directionality but do not require high fidelity frequency response.

The enclosure in Figure 8, which may be a full enclosure or a vented enclosure, allows part of the sound to escape in the backward direction. Without the horn, air molecules near the speaker would be free to move away from the speaker in any direction. The horn constrains air motion to the direction of the speaker axis. The sound waves expand as they move down the horn and are emitted most intensely in the cone defined by the horn.

SUMMARY

The fidelity of sound produced by any sound system depends not only on the electronics components and the speakers but also on the enclosures that house the speakers. Speaker enclosures

are designed to improve the frequency response of a system and to control the direction of sound energy.

DIRECTIONALITY OF SPEAKERS

MATERIALS

Audio signal generator.
Oscilloscope.
Speaker.
Full enclosure for speaker.
Microphone.
Audio amplifier.
Clip leads.
Protractor.
Sheet of white paper, 2 feet x 3 feet.
Speaker mount.
Straight edge, 2 feet long.

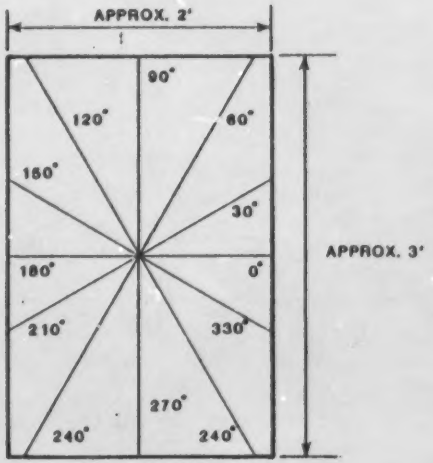


FIG. 9

PROCEDURE

1. Use a protractor and a straight edge to draw the lines as indicated in Figure 9, from center of a 2 foot x 3 foot sheet of paper.

2. Set up equipment, as shown in Figure 10, with speaker face parallel to the 0° - 180° line and the speaker dome above center of paper.

3. Turn audio signal generator on; set frequency selector to a speaker response peak in range from 500 Hz to 1000 Hz. Set output amplitude to a convenient level.

4. Turn oscilloscope on; set time base to 0.5 ms/cm; set voltage sensitivity to 1 V/cm.

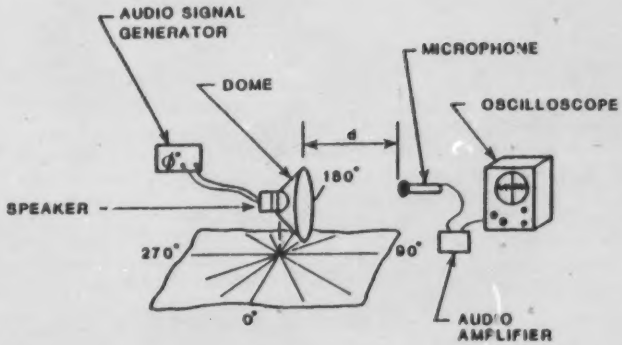


FIG. 10

5. Turn audio amplifier on; adjust audio amplifier gain and oscilloscope voltage sensitivity to obtain a signal with a height of about half of oscilloscope scale; make note of signal height.

6. Measure distance from center of paper (d) to microphone; note angular location of microphone and frequency of sound produced; make appropriate notations in data table.

7. Move microphone above next line on paper; move microphone along this line to point that gives same signal height; record new position and distance. Be sure speaker does not move from original setting (0° - 180° axis).

8. Continue above procedure around speaker.

9. Turn signal generator off; disconnect speaker. Do not change amplitude or frequency setting of signal generator.

10. Place speaker in enclosure.

11. Place enclosed speaker in original position on paper (0° - 180° axis); connect speaker wires; turn signal generator on.

12. Repeat all measurements for enclosed speaker at same frequency and amplitude.

13. Plot isointensity curves for open speaker (full line) and enclosed speaker (dotted line) on polar graph paper (use data from table). Choose an appropriate scale for radial distance that will allow isointensity plots to be drawn on the graph paper provided.

    Signal height _____ cm.

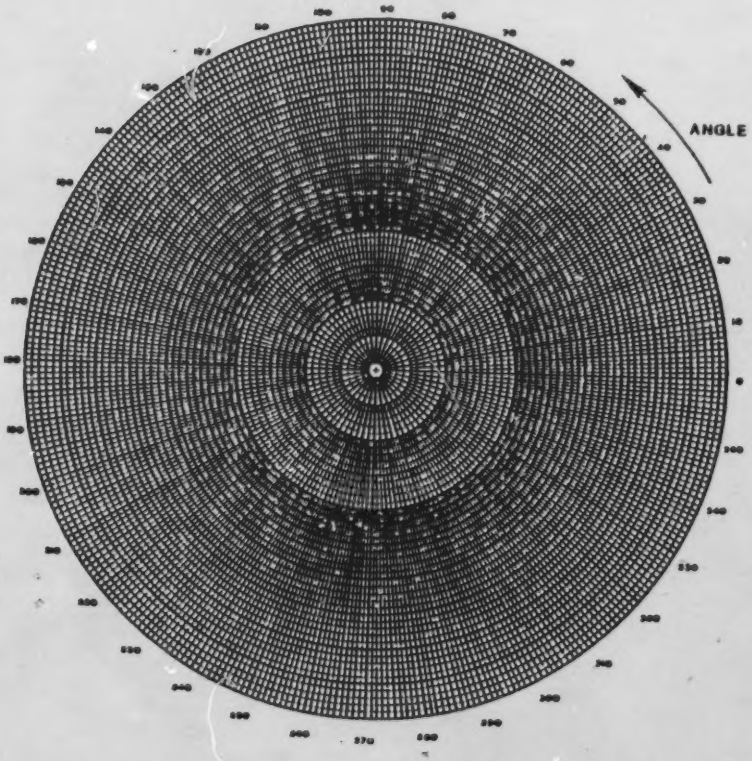
    Frequency _____ Hz.

14. Describe effects of the enclosure on speaker directionality.

DATA TABLE		
Angle (°)	Distance (d) (cm)	
	Open Speaker	Enclosed Speaker

Isointensity plot for 1) open speaker _____

2) enclosed speaker ----------



Frequency _____.

radial scale: 1 cm = _____.

SELF CHECK

1. Sketch a typical sound radiation pattern produced by a moving-coil speaker.

2. Draw and label a diagram showing how interference of reflected sound waves affects speaker performance.

3. Describe how horns control the directionality of speakers.

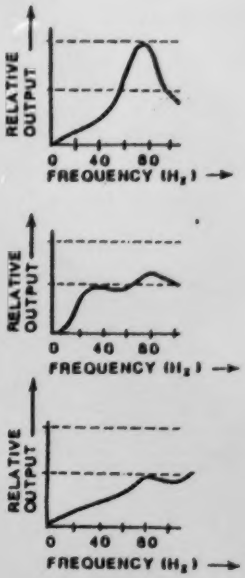
4. Describe how full enclosures improve speaker performance. Include the importance of acoustic absorption within the enclosure.

5. What is the major disadvantage of a full enclosure?

6. Describe how reflex enclosures improve speaker performance at low frequency.

7. The graphs at the right show frequency performance of a speaker with and without enclosures. Identify each graph as one of the following:

   a. Speaker with no enclosure.
   b. Speaker with full enclosure.
   c. Speaker with reflex enclosure.



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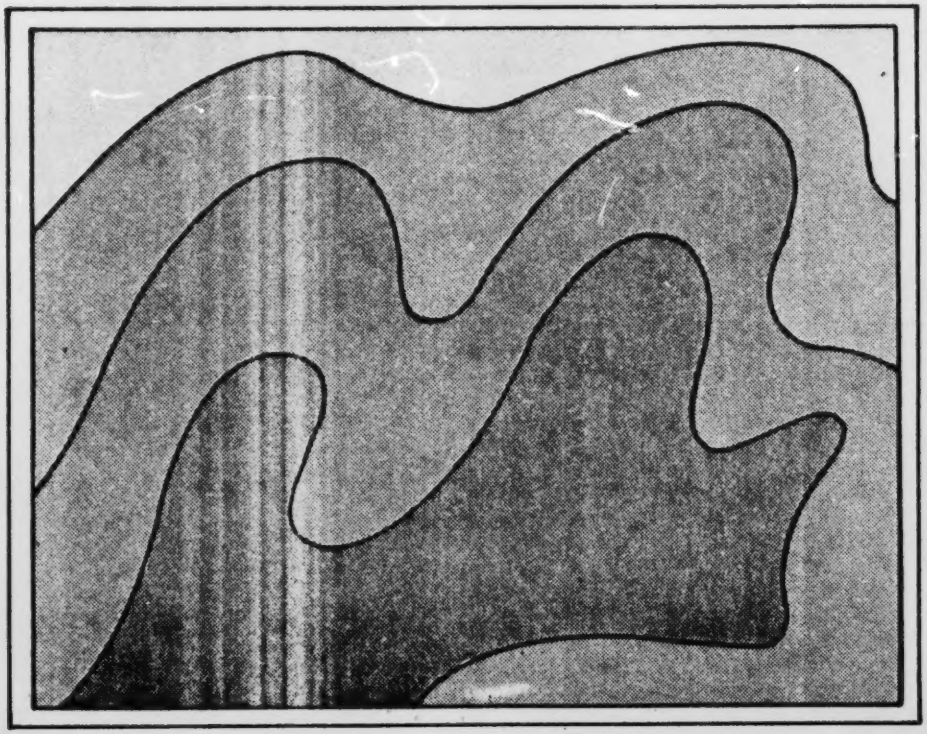
UNIFIED TECHNICAL CONCEPTS

MODULE 11 F 3

REVERBERATION TIME



FLUID APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

In the same way that a rubber ball bounces off a concrete wall, sound waves also bounce off the walls of a room. They are reflected sound waves, commonly called "echoes." Acoustics is a term used in the science of sound as it relates to hearing.

Some buildings are carefully designed for good acoustics. Because of the acoustical design of the Mormon Tabernacle in Salt Lake City, a pin dropped at a certain place in the auditorium can be clearly heard at the other end of the room. In the Statuary Hall at the United States Capitol in Washington, D.C., it is possible to stand at a certain spot in the building and distinctly hear a whisper uttered by someone at certain spots on the other side of the room, although the whisper may not be heard at points in between.

In this module, the student will learn the basic principles of acoustics as related to reverberation time and how to calculate acoustical properties of a room.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves."

OBJECTIVES

To complete this module, the student should be able to:

- Define:
  - Decibel.
  - Echo.
  - Reverberation.
  - Reverberation time.,
  - Acoustics.
  - Absorption coefficient.

Intensity.
Absorbing power.
Sabin.

- Calculate reflector distance given the time required for an echo to return to the source.
- Calculate reverberation time of a room given information about the room characteristics.
- List four major items that contribute to optimum reverberation time when a room does not have good acoustics and tabulate their respective absorbing powers in sabins.

SPEED OF SOUND

To travel, sound must have air or some other kind of a medium through which to pass. Sounds that we hear are normally transmitted through the air to our ears, but any elastic substance will transmit sound. The speed of sound in various mediums is given in Table 1.

TABLE 1. SPEED OF SOUND

Medium	Speed of Sound	
	m/s	ft/s
Air (0° C, 32° F)	331	1086
Air (25° C, 77° F)	343	1128
Iron	5210	17390
Brick	3660	11980
Oak	3850	12620
Water	1450	4700

The loudness of sound is referred to technically as the intensity of sound. The intensity of sound is measured in terms of a unit called the decibel, one tenth of a bel, named in honor of Alexander Graham Bell, inventor of the telephone. The intensity in decibels of several common sounds is given in Table 2.

TABLE 2. INTENSITY OF SOUND

Type of Sound	Intensity (decibels)
Whisper	11
Average dwelling	23
Average office	37
Ordinary conversation	55
Busy street	63
Heavy street traffic	78
Elevated train	90
Airplane engine	109

Reflected sounds, called echoes, are sounds that bounce off distant objects and return to the ear sometime after the original sound was made. The human ear is able to distinguish two sounds as being distinctly separate only if they reach the ear at least 1/10 second apart. In large rooms or auditoriums, the same sound reflected off walls can reach the ear at different times, making the sound garbled and difficult to understand. Such multiple echoes are called reverberations. The time it takes for the echoes to diminish so that they are no longer audible to the human ear is called the reverberation time. Reverberation time is therefore important in planning the acoustics of a room, theater auditorium, or test chamber in which clear sound is important.

It is possible to calculate distances by producing a sound and noting the time required for the sound to be reflected back to the listener or pickup device. Reflected sounds are used for example to determine the depth of a drilled well, and the distance to a mountaintop or to an atmospheric storm. The fathometer (depth finder) on a ship emits a high-pitched sound and uses the time required for sound to bounce back from the bottom of the sea to compute distance to the bottom, to a submerged wreck, or to distant fish shoals. The formula for determining distance by using echoes is derived by rearranging the definition for velocity (velocity = distance/time). The formula is given in Equation 1.

$$\text{Distance} = \text{velocity} \times \text{time}. \qquad (1)$$

Example A shows how the formula is used to solve a problem.

Similar exploratory soundings are made through soil by oil companies to locate rock formations that might hold oil and natural gas. Some waves will be reflected wherever there is a sudden change in composition of the earth's crust.

ABSORPTION OF SOUND

A sound in a room will continue to echo off the walls of the room until the sound is absorbed and becomes inaudible. Porous materials such as drapes, carpets, wall materials and clothing absorb sound waves, and smooth, hard surfaces reflect sound waves. The absorption coefficient of a material is that fraction of the sound energy a material will absorb at each

EXAMPLE A: ECHO PROBLEM

Given: Personnel on a navy ship found that an interval of 3.6 seconds elapsed between sending a sound wave pulse to the ocean bottom and receiving the reflected wave. (Velocity of sound in seawater is 1450 m/s.)

Find: Ocean depth.

Solution: Distance = velocity x time

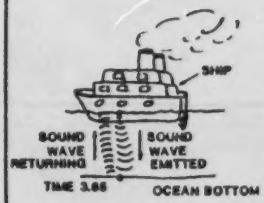
$$= 1450 \frac{m}{\cancel{s}} \times 3.6 \cancel{s}$$

Distance = 5220 m (distance the sound traveled).

The sound traveled 5220 m. The bottom is one-half of this distance, because sound travels both ways.

$$\text{Depth of ocean} = \frac{5220 \text{ m}}{2}$$

Depth of ocean = 2610 meters at this location.



reflection or transmission. Absorption coefficients of some common materials are given in Table 3.

TABLE 3. ABSORPTION COEFFICIENTS OF COMMON MATERIALS

Material	Absorption Coefficient (β)
Acousti-celotex tiles	0.82
Carpet, heavy	0.40
Chalkboard	0.06
Draperies	0.50
Glass	0.025
Linoleum	0.03
Plastered wall, paneling, wood floor, brick	0.04
Open window	1.00

Absorption coefficients can be used to calculate the absorbing power of materials in a room. The unit for absorbing power is the sabin. The absorbing power of one sabin is defined as the absorption of 1 ft^2 of open window. The total absorbing power (a) of all surfaces and materials in a room is equal to the sum of the absorption coefficients (β) for every surface in the room, multiplied by the respective area (A) of each surface. The formula for the absorbing power of a room is given in Equation 2.

$$a = \beta_1 A_1 + \beta_2 A_2 + \beta_3 A_3 + \ldots \qquad (2)$$

The absorbing power of some irregular objects is given in Table 4.

TABLE 4. ABSORBING POWER

Object	Absorbing Power (Sabins)
Audience, each person	4.0
Empty chair, wood	0.3
Empty chair, upholstered	2.0

ACOUSTICS OF A ROOM

A room has good acoustics when speech can be heard equally well throughout the room, without troublesome reverberations and without "dead" spots due to interference. Interference can be minimized by shaping the room to be free from pillars, overhangs and the like. Reverberation time can be kept at acceptable levels by using sound-absorbing materials as needed in the room.

Optimum reverberation time for small auditoriums is from 0.5 to 1.0 second. For large auditoriums, a reverberation time of up to 2.0 seconds is acceptable. If the reverberation time of a room is too long, echoes from previous syllables uttered by the speaker will arrive at the listener's ear at a time when they will interfere with the ear's reception the next syllable. Architects are therefore concerned with reverberation time in the design of a building, especially in the case of public

auditoriums or concert halls. The formula for calculating reverberation time is given in Equation 3.

$$\text{Reverberation time} = 0.05 \frac{\text{volume of room}}{\text{absorbing power in room}}.$$

$$\text{In symbols:} \quad t = 0.05 \frac{v}{a} \qquad (3)$$

where

t = reverberation time.
v = volume of room.
a = absorbing power in room from Equation 2.

Example B shows how Equations 2 and 3 are used to calculate reverberation time in a room.

EXAMPLE B: REVERBERATION TIME CALCULATION FOR A ROOM

Given: A small auditorium, 70 ft by 95 ft by 15 ft high. Areas of important surfaces and objects within the building are: drapes, 200 ft^2; plaster, 8050 ft^2; paneling, 3000 ft^2; linoleum, 5650 ft^2; heavy carpet, 1000 ft^2, acousti-celotex, 2650 ft^2. In addition the room is filled with 90 empty upholstered chairs and 120 people.

Find: Reverberation time of the room.

Solution: a. Calculate absorbing power of each item in the room.

Item	B & A	Absorbing Power (Sabins)
Drapes	200 x 0.50	100
Plaster	8050 x 0.04	322
Paneling	3000 x 0.04	120
Linoleum	5650 x 0.03	169.5
Carpet	1000 x 0.40	400
Acousti-celotex	2650 x 0.82	2173
Audience	120 x 4.0	480
Empty upholstered chairs	90 x 2.0	180
	Total absorbing power a = 3944.5 sabins.	

b. Calculate reverberation time:

$$\text{Reverberation time} = 0.05 \frac{\text{volume of room}}{\text{absorbing power in room}}$$

$$t = 0.05 \frac{70 \text{ ft} \times 95 \text{ ft} \times 15 \text{ ft}}{3944.5 \text{ sabins}}$$

$$t = 1.26 \text{ seconds.}$$

c. The acoustics of the room are evaluated as follows:

With a reverberation time of 1.26 seconds, the auditorium will not have good acoustics. Reverberation time for a small auditorium should be between 0.5 and 1.0 seconds. More drapes, carpeting, acousti-celotex or more people in the audience will bring the value to an acceptable level.

LABORATORY

MATERIALS

Small auditorium or classroom for which acoustical properties can be calculated.
Measuring tape.

PROCEDURE

1. Measure room dimensions, windows, doors and other items involved, such as drapes, blackboard, etc.
2. Calculate reverberation time of room.
3. Determine whether or not acoustics of room are satisfactory for speech.
4. Make recommendations on how reverberation time of this room could be adjusted.

SELF CHECK

1. Define:
   - Decibel.
   - Echo.
   - Reverberation.
   - Reverberation time.
   - Acoustics.
   - Absorption coefficient.
   - Intensity.
   - Absorbing power.
   - Sabin.
2. With a depth-sounding apparatus, it was found that 4.6 seconds elapsed between transmission of a sound pulse and recept of its echo. If velocity of sound in water is 1450 m/s, what was depth of the water?
3. In a classroom 38 ft by 42 ft by 10 ft high, there are 240 ft^2 of glass, 200 ft^2 of paneling, 300 ft^2 of chalkboard, 1200 ft^2 of plaster; the ceiling is covered with plaster and the floor with linoleum tile. There are 45 wood chairs (empty) in the room and 50 more are occupied by students. Find the reverberation time of the room with and without the students.

4. Calculate the area of acousti-celotex tile required to reduce reverberation time of the room described in the previous problem to 0.8 second, when students are present.
5. List four major items that contribute to optimum reverberation time when a room does not have good acoustics; tabulate their respective absorbing powers in sabins.

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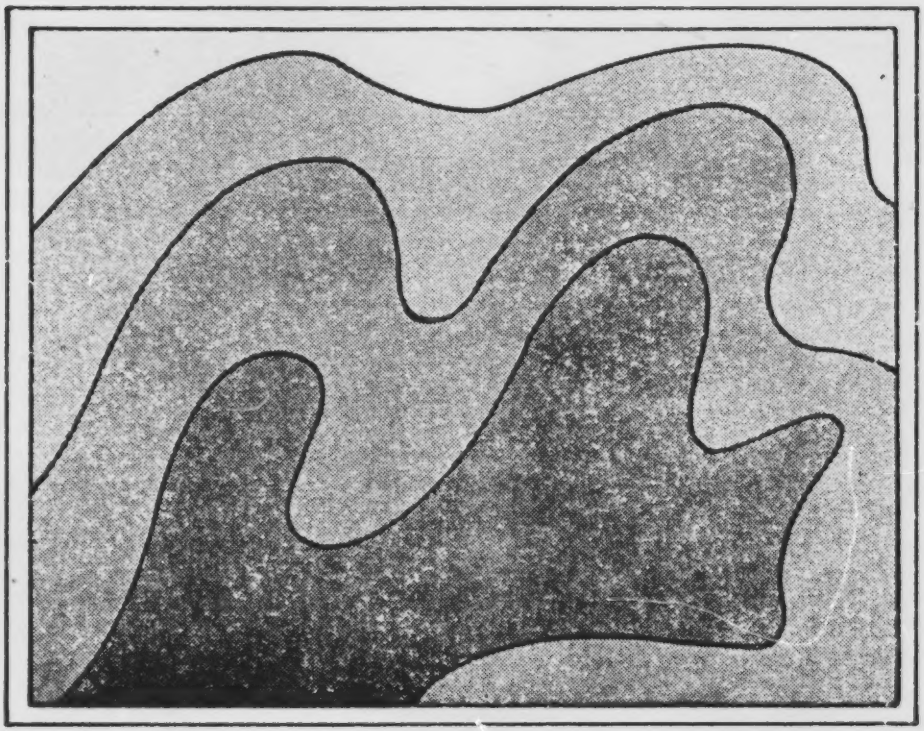
UNIFIED TECHNICAL CONCEPTS

MODULE 11 F 4

DOPPLER EFFECT



FLUID APPLICATION OF VIBRATIONS AND WAVES,

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

The pitch of the whistle of a locomotive sounds higher when it is approaching then when it is receding from a stationary listener. This phenomenon was explained by an Austrian named Christain Johann Doppler in 1842 and applies, not only to sound waves, but to waves in general. For example, the color of a luminous body must change due to its motion relative to an observer.

The Doppler effect has proved to be, not only an interesting physical phenomenon, but also a useful one. It has enabled astronomers to determine the relative speeds of galaxies, enriching their insight into the origin and future of the universe. Almost everyone is familiar with the defense and traffic-control applications of radar, which operates on the Doppler principle. The production of shock waves by supersonic aircraft is another common occurrence accompanying the Doppler effect.

In this module, the Doppler effect is presented for both sound and electromagnetic waves, including appropriate equations for determining the frequency shifts that result. The phoenomenon of shock waves, and applications to astronomy, spectroscopy, and radar are discussed. In the laboratory, the student will measure the Doppler frequency of sound waves.

PREREQUISITES

The student should have completed Concept Module II-0, "Vibrations and Waves," and Application Module IIM1, "Speed of Sound in Materials."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Calculate the Doppler frequency of a sound wave, given speeds of the source and observer and the frequency if both are at rest.

- Explain how beats are produced; define "beat frequency."
- Draw a diagram of a shock wave, and explain how it is produced; define "Mach cone" and "Mach number"; calculate the angle included in the Mach cone.
- Calculate the Doppler frequency of electromagnetic radiation, given the frequency of the source and its relative speed.
- Explain how radar is used to track a moving object, and determine its speed; calculate the velocity of a moving object relative to a radar transmitter, given the frequency and Doppler frequency.
- Explain the Doppler broadening of absorption and emission spectral lines.
- Calculate the relative velocity of a luminous heavenly body, given the Doppler shift of one of its spectral lines.
- Given the appropriate equipment, measure the Doppler shift of sound waves.

DISCUSSION

Figure 1 depicts sound waves produced by a stationary source S and perceived by a stationary observer at O. The concentric circles represent wave fronts. The frequency of the sound heard by the observer is the same as that heard if he or she were positioned at the source. Wave frequency is indicated by Equation 1.

$$f = \frac{v}{\lambda} \qquad (1)$$

where

f = frequency of source.
v = velocity of waves.
λ = wavelength.

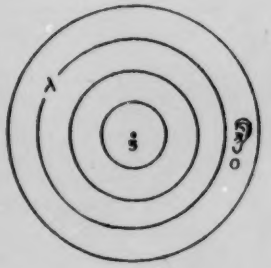


FIG. 1

On the other hand, if the observer moves toward the source at velocity v_o as shown in Figure 2, he encounters each wavefront sooner than it normally would reach him if he were stationary. The wavefronts perceived by the moving observer appear to be more closely spaced, thereby encountered at a higher frequency. The observer, therefore, hears a higher pitch than when he is stationary.

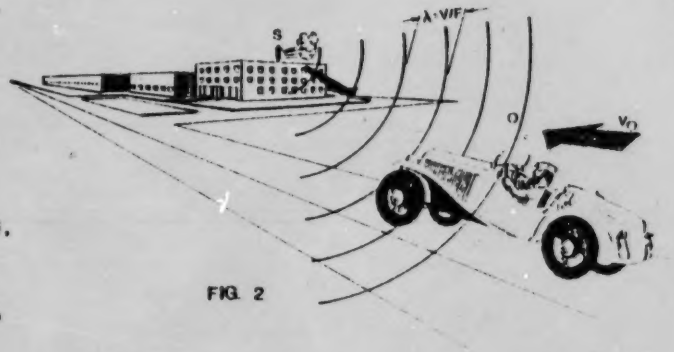


FIG. 2

The number of waves received by the stationary observer in time t is vt/λ. When he moves toward the source, he encounters the waves at a higher rate. He must receive an additional $v_o t/\lambda$ waves in the same time interval t. The frequency f_o heard by the observer is the number of waves received in time t divided by t, as in Equation 2.

$$f_o = \frac{\frac{vt}{\lambda} + \frac{v_o t}{\lambda}}{t}$$

$$= \frac{v}{\lambda} + \frac{v_o}{\lambda} \qquad \text{(Dividing by t)}$$

$$= \left(\frac{v}{\lambda}\right)\left(1 + \frac{v_o}{v}\right) \qquad \text{(Factoring)}$$

$$f_o = f_s\left(1 + \frac{v_o}{v}\right) \qquad \text{(Since } f = v/\lambda\text{)} \qquad (2)$$

where

f_o = frequency of waves heard by observer.
f_s = frequency of waves at source.
v = velocity of sound in air.
v_o = velocity of observer.

The frequency f_o heard by the observer is the frequency f_s heard if $v_o = 0$ (observer stationary), plus the increase fv_o/v due to the motion of the observer. The frequency f_o is called the "Doppler frequency."

If the observer is moving away from the source, he receives $v_o t/\lambda$ fewer waves in time t. The Doppler frequency is then given by Equation 3.

$$f_o = f_s\left(1 - \frac{v_o}{v}\right) \qquad (3)$$

The listener moving away from the source hears a lower pitch, or frequency, than he would if he were stationary. Example A illustrates how the "Doppler shift" — or change in frequency, $(f_o - f_s) = (-f_s\, v_o/v)$ — is found for a moving observer.

EXAMPLE A: DOPPLER EFFECT FOR A MOVING OBSERVER

Given: An automobile traveling at 27 m/s approaches a factory sounding its whistle at a frequency of 1980 Hz.

Find: The frequency heard by the motorist (a) as he approaches and (b) after he passes the factory and moves away.

Solution: The velocity v_o of the observer is 27 m/s, and that of the wave is 330 m/s, the velocity of sound in air. On approach, the driver hears —

$$f_o = f_s\left(1 + \frac{v_o}{v}\right)$$

$$= (1980\ \text{Hz})\left(1 + \frac{27}{330}\right)$$

$$f_o = 2142\ \text{Hz}$$

As he moves away he hears —

$$f_o = f_s\left(1 - \frac{v_o}{v}\right)$$

$$= (1980\ \text{Hz})\left(1 - \frac{27}{330}\right)$$

$$f_o = 1818\ \text{Hz}$$

The Doppler shift in either case is $f_o - f_s = \pm 162$ Hz

In the case of the moving observer and stationary source, the actual wavelength $\lambda = v/f$ of the sound waves does not change. If, however, the observer remains stationary and the source moves toward him at a speed v_s as in Figure 3, the wave fronts received by the observer are closer together. This effect occurs because with each new wave emitted by the source, the source is closer to the observer. The wavelength is, therefore, shortened by the distance v_s/f_s traveled by the source in each cycle. The Doppler frequency f_o is found in Equation 4 developed below.

$$f_o = \frac{v}{\lambda_o}$$

$$= \frac{v}{(\lambda - \frac{v_s}{f_s})} \quad \text{(Since } \lambda_o = \lambda - v_s/f_s\text{)}$$

$$= \frac{f_s v}{\lambda f_s - v_s} \quad \text{(Simplifying the fraction)}$$

$$f_o = \frac{f_s v}{(v - v_s)} \quad \text{(Since } v = \lambda f_s\text{)} \qquad (4)$$

where

λ_o = wavelength received by observer as source approaches.

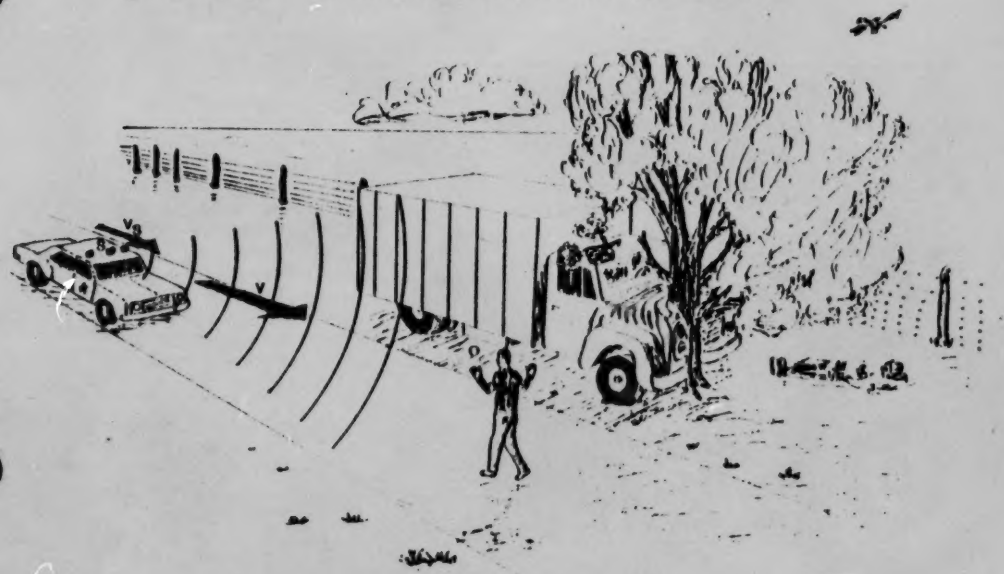
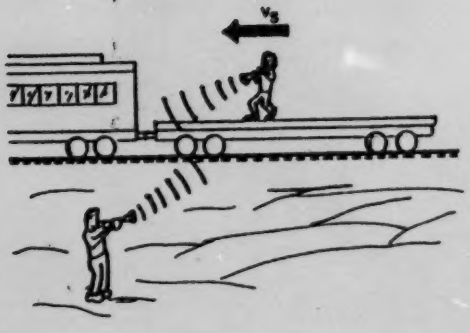


FIG. 3



(a)

Since the waves received are closer together, the frequency heard by the observer is higher than if the source were stationary. When the source moves away from the observer, the wavelength is now larger by v_s/f, and the Doppler frequency becomes that indicated by Equation 5. In this equation, f_o is less or lower in pitch than f_s.

$$f_o = \frac{f_s v}{(v + v_s)} \qquad (5)$$

The Doppler effect for sound was first tested experimentally in 1845 by Buys Ballot. A trumpet player blew his horn while riding on a flatcar pulled by a locomotive, as in Figure 4a. A second trumpet player stood beside the tracks and blew his horn simultaneously at the same frequency. The sound waves from the two trumpets are illustrated in Figure 4b. Notice that the frequency of the moving trumpet is increased slightly as it approaches the stationary player. The waves are shown plotted together in Figure 4c. Since the waves interfere, they can be added together to give a resultant wave like the one shown in Figure 4d. The maxima of such a resultant wave are called "beats." The beat frequency is the difference between the frequencies of the two interfering waves, and is the Doppler shift in frequency.

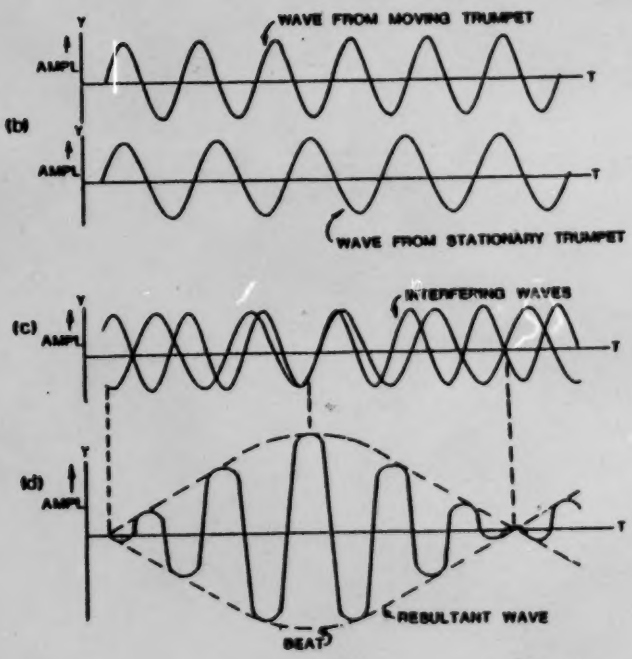


FIG. 4

Examples B and

C illustrate the Doppler shift for a moving source.

EXAMPLE B: DOPPLER EFFECT FOR A MOVING SOURCE

Given: A trumpet player blowing his horn at 440 Hz approaches at a speed of 3 m/s a stationary player blowing his horn at the same frequency.

Find: The Doppler shift in frequency.

Solution: The speed of the source v_s is 3 m/s.

$$f_o = \frac{f_s v}{(v - v_s)}$$

$$= \frac{(440 \text{ Hz})(330)}{(330 - 3)}$$

$$f_o = 444 \text{ Hz}$$

The Doppler shift or beat frequency is –

$$f_o - f_s = 444 - 440 = 4 \text{ Hz}$$

EXAMPLE C: DOPPLER EFFECT FOR A MOVING SOURCE

Given: A siren emitting sound of frequency 1000 Hz moves away from a stationary observer at 40 m/s.

Find: The frequency heard by the observer.

Solution: For receding sources, the pitch or frequency is lower.

$$f_o = \frac{f_s v}{(v + v_s)}$$

$$= \frac{(1000 \text{ Hz})(330)}{330 + 40}$$

$$f_o = 892 \text{ Hz}$$

The mathematical results of the Doppler effect for sound can be summerized by Equation 6 for all cases considered.

$$f_o = \frac{f_s(v \pm v_o)}{(v \pm v_s)} \qquad (6)$$

Equation 6 allows treatment of cases in which both the source and the observer are in motion. The appropriate choice of sign must be memorized because the frequency f_o must be higher than f_s for the converging source and observer and must be lower for the receding source and observer. Cases in which both the source and the observer are moving are presented in Examples D and E.

EXAMPLE D: DOPPLER SHIFT FOR BOTH OBSERVER AND SOURCE MOVING

Given: An automobile traveling at 20 m/s due east approaches another traveling at 25 m/s due west. The westbound driver sounds his horn, which has a frequency of 1200 Hz.

Find: The frequency heard by the eastbound motorist.

Solution: The Doppler frequency is higher than 1200 Hz since the cars are approaching each other. Thus, one chooses the positive sign for the numerator and the negative sign for the denominator.

$$f_o = \frac{f_s (v + v_o)}{(v - v_s)}$$

$$= \frac{(1200 \text{ Hz})(330 + 20)}{(330 - 25)}$$

$$f_o = 1377 \text{ Hz}$$

EXAMPLE E: DOPPLER SHIFT FOR BOTH OBSERVER AND SOURCE MOVING

Given: A car with a siren of frequency 2000 Hz traveling at 30 m/s is chasing an observer who is moving at 35 m/s.

Find: The frequency of the siren heard by the observer.

Solution: The observer moves away from the source, and the source moves toward the observer.

$$f_o = \frac{f_s (v - v_o)}{(v - v_s)}$$

$$= \frac{(2000 \text{ Hz})(330 - 35)}{(330 - 30)}$$

$$f_o = 1967 \text{ Hz}$$

SHOCK WAVES

When the wave source moves through a medium at a faster rate than the speed of the wave in the medium, the wavefront becomes conical in shape, with the moving source at its apex (Figure 5). Examples of this type of motion are the bow wave of a boat and the shock wave from a supersonic aircraft or from a bullet. The waves pile up at the surface of the cone, causing a severe pressure disturbance. The angle θ between the velocity v_s of the source and the surface of the cone is given by Equation 7 and illustrated by Figure 5.

$$\sin \theta = \frac{vt}{v_s t} = \frac{v}{v_s} \qquad (7)$$

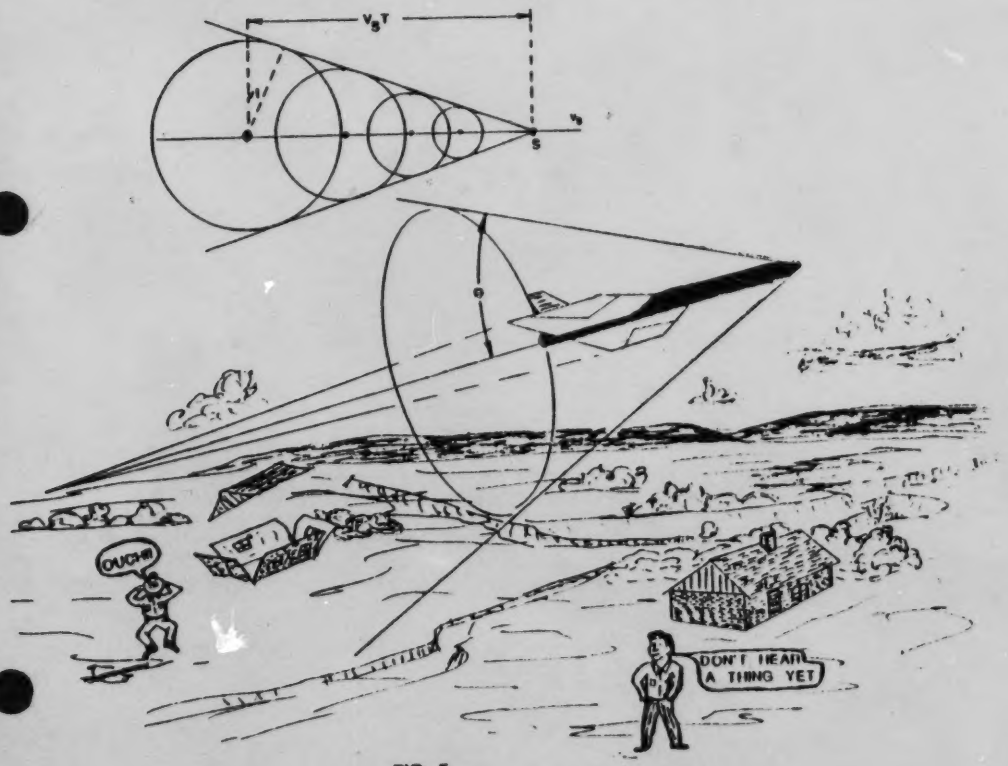


FIG. 5

The cone produced by a supersonic aircraft is called the "Mach cone," and the ratio v_s/v the "Mach number," after the physicist Ernst Mach. When an aircraft "breaks the sound barrier," it is traveling faster than 330 m/s, and the surface of the Mach cone which reaches observers on the ground, causes them to hear a sharp crack. The cone reduces to two intersecting lines for a bow wave.

Example F shows the calculation of the angle of a shock wave.

EXAMPLE F: SHOCK WAVES

Given: A destroyer traveling at 20 knots (1 knot = 1.15 miles/hour) generates waves traveling at 8 knots.

Find: The angle θ between the ship's direction and its bow wave.

Solution: From Equation 7 –

$$\theta = \sin^{-1}\left(\frac{v}{v_s}\right)$$

$$= \sin^{-1}\left(\frac{8}{20}\right)$$

$$\theta = 23.6°$$

THE DOPPLER EFFECT FOR ELECTROMAGNETIC WAVES

One might expect that a moving light source would appear to be changed in color, just as a moving sound source appears to be changed in pitch; however, the equations developed for sound do not apply. The velocities v_o and v_s were defined with respect to a particular medium in which sound waves travel at speed v. Sound, being a pressure wave, requires such a medium for transmission. No such medium of transmission is required for electromagnetic waves, which can travel through a vacuum. This does not mean there is no Doppler shift for light waves; rather, it implies that only the speed of the source relative to the observer can have any significance.

For nonrelativistic speeds ($u \ll c$), the Doppler frequency is calculated by use of Equation 8.

$$f_o = f_s \left(1 \pm \frac{u}{c}\right) \qquad (8)$$

where

u = relative motion of source and observer.
c = speed of light in a vacuum.
f_s = frequency of the source.
f_o = Doppler-shifted frequency as detected by observer.

The choice of sign is determined just as for sound waves.

The Doppler effect for radio waves, which have wavelengths that extend from a few centimeters up to 10^5 meters, is used for radar traffic control and defense systems. Radar permits measurement of the beat frequency between emitted radio waves and the reflected wave from an approaching or receding object to determine its speed (Figure 6). Example G illustrates how radar is used in traffic control.

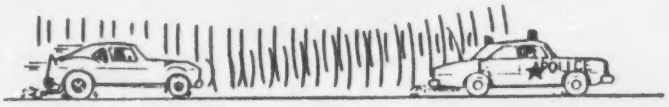


FIG. 6

EXAMPLE G: DOPPLER EFFECT FOR RADIO WAVES

Given: Radio waves having a frequency of 3×10^9 Hz (3GHz) are used to measure the speed of an approaching motorist on the highway. The beat frequency measured is 300 beats/s.

Find: The speed of the approaching automobile.

Solution: The beat frequency is equal to the Doppler shift in frequency, $(f_o - f_s)$.

$f_o = f_s\,(1 + u/c)$ (used + sign since auto is approaching)

$f_o - f_s = f_s\,(\frac{u}{c})$ (rearranged Equation 8)

$$300 = (3 \times 10^9 \text{ Hz})\left(\frac{u}{3 \times 10^8 \text{ m/s}}\right)$$

$$u = \frac{(300)(3 \times 10^8 \text{ m/s})}{3 \times 10^9}$$

$u = 30$ m/s, or about 67 mph

The shift in wavelength that results from the Doppler effect for electromagnetic waves for nonrelativistic relative speeds u is $\lambda_o(\pm u/c)$. Since c is 3×10^8 m/s, a change in color is not observable to a motorist approaching a traffic light; the shift for a typical automobile speed of 20 m/s and light of wavelength 6000 A° is 6000 A° x 20 / (3×10^8), or 4×10^{-4} A°, which is 7×10^{-6}%!

The Doppler shift in wavelength becomes significant at higher relative speeds u. Gas molecules heated by an electric discharge emit light of wavelengths characteristic of the gas. Because the molecules are very energetic, they have large velocities. Doppler shifts occur in their spectra, the magnitude and direction of the shift depending upon the speed of the individual molecule producing a particular spectral line, and upon whether or not it is moving toward or away from the photographic plate. Similar shifts occur in the absorption spectra from such molecules when white light shines through the gas and when characteristic wavelengths are selectively absorbed. The resulting absorption-and-emission spectral lines, representing many molecules, are broadened. Figure 7 illustrates a method of viewing such a spectrum for sodium vapor.

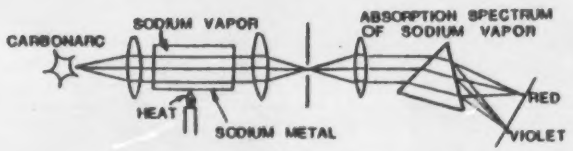


FIG. 7

Absorption spectral lines from a star are not only broadened because of random molecular motions, but also are shifted considerably because of the relative motion of the star with respect to the earth. Such shifts in wavelengths are used to determine the velocities of stars and of entire galaxies relative to the earth. Science has determined that all the distant galaxies that have been studied are moving away from the earth with speeds proportional to their distances; this finding establishes the model of the universe as an expanding one and suggests a primal "big bang" as the origin of the expansion.

Example H consists of the solution to a problem involving the relative speed of a galaxy.

EXAMPLE H: RELATIVE SPEED OF A GALAXY

Given: The Doppler wavelength of a spectral line known to be of wavelength 6560 A° is measured as 6586 A°.

Find: Velocity of the galaxy relative to the earth.

Solution: The Doppler wavelength of the spectral line is increased by 6586 - 6560 = 26 A°. The galaxy, therefore, is receding. The velocity is found as follows:

$$26\ A^\circ = 6586\ A^\circ \frac{u}{(3 \times 10^8\ m/s)}$$

$$u = \frac{(26)(3 \times 10^8\ m/s)}{6586} \quad \text{(solving for u)}$$

$$u = 1.2 \times 10^6\ m/s$$

SUMMARY

The Doppler effect is the apparent change in the frequency of sound or of electromagnetic radiation due to relative motion between the source and the observer. The pitch, or frequency, of the sound emitted by a moving object (e.g. the whistle of a moving train) appears to a stationary observer to increase as the object approaches him and to decrease as it recedes from him. If the source is stationary, an observer moving toward it intercepts more sound waves per unit time than if he were stationary; consequently, he hears a higher frequency. When the observer moves away from the source, fewer of the waves catch up with him during a unit time, and he hears a lower frequency.

The Doppler effect is utilized widely in the area of electromagnetic radiation. The light emitted by a moving object appears more red (red light being of lower frequency than the other colors) when it is receding from the observer (or the observer receding from it). Thus, the fact that the light emitted by the stars of distant galaxies experiences a shift when observed from the earth is taken to mean that these distant galaxies are receding from our galaxy. This is the principal evidence for the widely-accepted hypothesis concerning the expansion of the universe. The Doppler effect also is used in radar to distinguish between stationary and moving targets and to provide information concerning their velocity, by measuring the frequency shift between the emitted and the reflected radiation.

DOPPLER SHIFT OF SOUND WAVES

MATERIALS

Stroboscope.
Oscilloscope.
Microphone.
2500-rpm electric drill.
Vise.
One of the following (1 or 2):

1. Bolt and nut.
   Meter stick cut to 30 cm in length.
   Sound source (tuning fork and rubber mallet or oscillator circuit with speaker).
   Counterweight and clamps.
2. Doppler's Principle Apparatus (Sargent Welch 0964) or equivalent and compressed air.

PROCEDURE

If Doppler's Principle Apparatus is available, omit Step 1.

1. Mount the meter stick on the drill, using the nut and bolt in such a manner that it turns like a propeller. (See Figure 8.) Mount the sound source at one end of the meter stick, balanced by a counterweight at the opposite end if necessary. Be sure that both the source and the counterweight are clamped securely.

2. Determine frequency of stationary sound source as follows: Adjust oscilloscope so that the sound source produces a clear sine wave on the screen (Figure 8). Record the seconds per division (SEC/DIV) and the number of divisions in one cycle in Data Table. The period T of the wave is the number of divisions in one cycle, multiplied by SEC/DIV. Frequency is the inverse of the period T: $f = 1/T$. Calculate f and record in Data Table

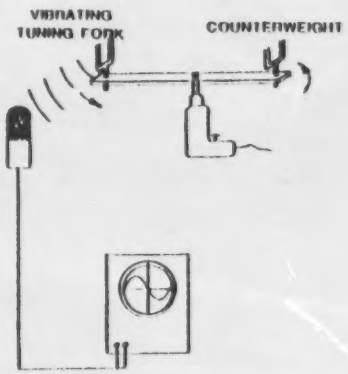


FIG. 8

under RESULTS.

3. With the sound source rotating, determine the minimum and maximum periods, T_{min} and T_{max}, as in procedure 2; and record data and results. The Doppler frequencies for approach and recession are found as follows:

$$f_A = 1/T_{min} \quad \text{(approach)}$$

$$f_R = 1/T_{max} \quad \text{(recession)}$$

Calculate and record these values.

4. Determine velocity v_s of source as follows: Measure and record radius of rotation r in Data Table. Use stroboscope to determine the angular velocity ω, or use manufacturer's value for the drill if the stroboscope is unavailable. Record ω in the Data Table. Convert ω to rad/s and record. Calculate v_s from the equation $v_s = r\omega$ and record.

5. Calculate the Doppler frequencies for approach f_A and recession f_R from source speed v_s found in Step 4 and stationary source frequency found in Step 2; use the following equations:

$$f_A = f\ v\ /\ (v - v_s) \quad \text{(approach)}$$

$$f_R = f\ v\ /\ (v + v_s) \quad \text{(recession)}$$

Record these values, and compare them with those found by Step 3.

6. What is "Doppler shift"?

DATA TABLE

Frequency measurments:

Stationary source: SEC/DIV = _____
DIV/CYCLE = _____
Rotating Source: SEC/DIV = _____
DIV/CYCLE = _____
Determination of v_s: r = _____
ω = _____

RESULTS:

Stationary source: T = _____ s
f = _____ Hz
Rotating source:

Experimental values:
T_{min} = _____ s f_A = _____ Hz

T_{max} = _____ s f_R = _____ Hz

Theoretical values:

ω = _____ rad/s f_A = _____ Hz

v_s = _____ m/s f_R = _____ Hz

SELF CHECK

1. Find the Doppler frequency of an automobile radio tuned to 1500 x 10^3 Hz (1500 KHz) if (a) the automobile is stationary and the observer approaches at 30 m/s and if (b) the observer is stationary and the automobile approaches at 30 m/s.

2. Find the Doppler frequency heard by a motorist traveling at 20 m/s east, away from the whistle of a locomotive traveling at 75 m/s west if the whistle frequency is 3000 Hz.

3. Explain how beats are produced, and define "beat frequency"; explain how radar speed control works.

4. Find the approach speed of an airplane detected with radar of frequency 3 x 10^9 Hz if the beat frequency is 990 Hz.

5. Draw a diagram of a shock wave and explain how it is produced; define "Mach cone" and "Mach number."

6. Find the angle included in the bow wave for a toy sailboat moving at 1 m/s if the bow waves have a speed of 25 cm/s.

7. Explain the Doppler broadening of emission-and-absorption spectral lines.

8. Find observed wavelength of a stellar spectral line of wavelength 6000 A° if the star is receding from Earth at u = 1.5 x 10^6 m/s.

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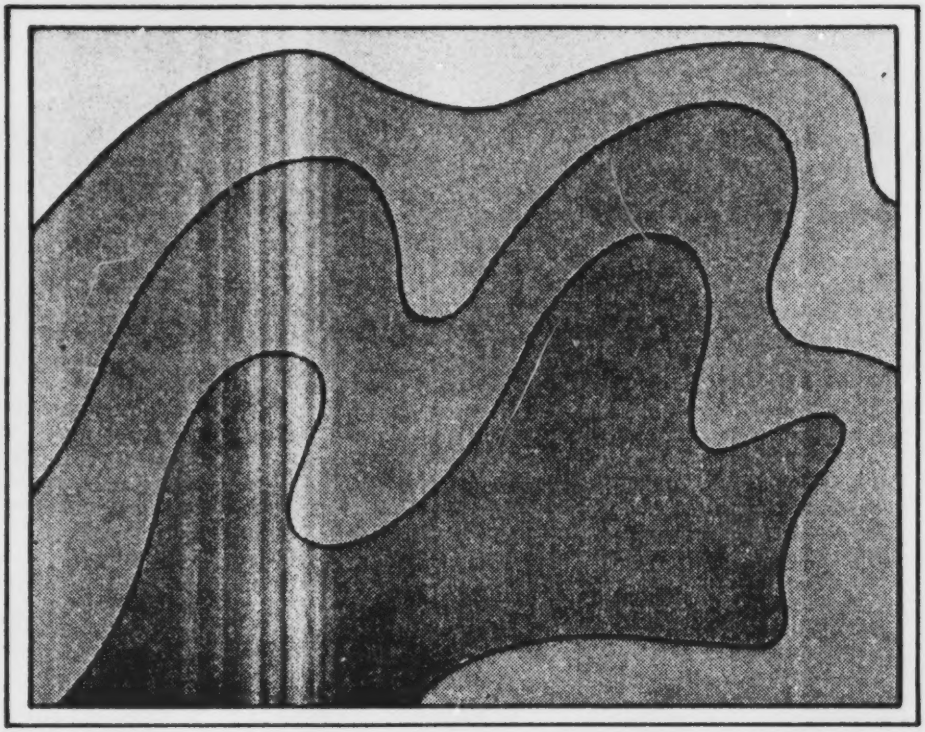
UNIFIED TECHNICAL CONCEPTS

MODULE 11 F 5

PRESSURE MEASUREMENT OF SOUND WAVES



FLUID APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

Air consists of a large number of molecules. These molecules can be thought of as very small spheres moving with random motion and distributed uniformly in space. Although we cannot see air, we know it exerts pressure; for example, a one-square-inch column of air extending from sea level to the top of the atmosphere would weigh 14.7 lb. This force per unit area (14.7 lb/in^2) normally is called "atmospheric pressure."

Sound waves are small variations in pressure superimposed upon the normal atmospheric pressure. These small changes from atmospheric pressure are called "sound pressure." This module examines the elementary principles that characterize the production, transmission, and measurement of sound. In the laboratory, the student will measure sound-pressure levels and will draw the contours of equal sound-pressure levels around a sound source.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define sound as a compression/rarefaction wave.
- Explain the decibel notation.
- List the advantages of using the decibel scale in sound-pressure measurements.
- Given the maximum pressure variations of a sound wave, calculate the sound level in decibels.
- Calculate the intensity level of a sound source, given its pressure level.
- Calculate the pressure level of combined sound sources, given all intensity levels.

- Given the appropriate equipment, measure the sound pressure levels in 5 different areas.
- Given the appropriate equipment, determine 3 contours of equal sound pressure around a noise source.

SOUND GENERATION AND PROPAGATION

Sound must have a medium in which to propagate. Sound commonly propagates through air, but it also can propagate through other gases, liquids, and solids. Unlike electromagnetic radiation, sound cannot travel through a vacuum. In any case, sound is a rapid variation of the pressure around a steady-state value at each point in the transmitting medium. This module is restricted to a discussion of sound propagation through air.

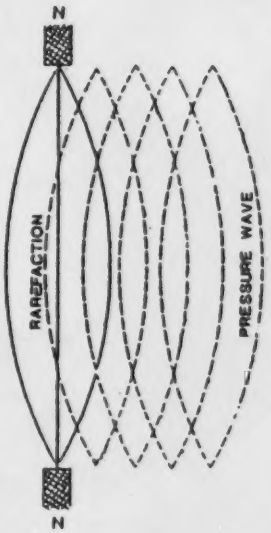


FIG. 1

The generation and propagation of sound through air is visualized easily by a simple example. Consider a plate suspended in air, as illustrated in Figure 1. When struck, the plate vibrates back and forth. As the plate moves, it compresses the air in the direction of its motion; and when it reverses direction, it leaves a partial vacuum, or rarefaction, of air behind it. These alternate compressions and rarefactions cause small but repeated fluctuations in atmospheric pressure that travel outward from the plate with the speed of sound. When these pressure variations strike an eardrum, they cause it to vibrate like the plate, in response to the changes in pressure. The disturbance of the eardrum is translated into a neuro-sensation and is carried to the brain, where it is interpreted as sound.

The elastic medium through which sound is transmitted behaves as if it were a succession of particles joined together by springs. The motion of one particle can be transmitted to others by pushing and pulling. If one of these particles is set in motion, its neighbor follows with the same motion, but lags behind the first in time.

A sound wave consists of a series of condensations and rarefactions. The condensations are regions of high sound pressure, and the rarefactions are regions of low sound pressure. These variations are impressed upon the normal, steady-state atmospheric pressure.

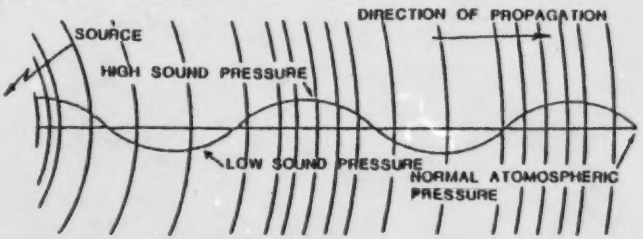


FIG. 2

The resulting motion of the medium is known as "wave motion." Although the condensations and rarefactions travel through the medium, the sound wave can be pictured by "freezing" the motion for an instant. Such a sound wave is illustrated in Figure 2. The wavy line drawn along the axis is a plot of pressure at different points along the wave.

PLANE AND SPHERICAL WAVEFRONTS

Sound wave propagation is treated most easily in terms of two simple forms — "plane waves" and "spherical waves." These names are derived from the geometric description of the wave front — the area occupied by the forward part of the pressure disturbance.

A plane wave results if the source is large, if the measuring point is a long distance from the source, and if there are no reflecting surfaces of any kind between the source and the measuring point. A wave of this type is shown in Figure 3a.

The spherical sound wave generally comes from a small, pulsating sphere if there are no reflecting surfaces in the vicinity of the source of the point of measurement. A spherical wave is illustrated in Figure 3b.

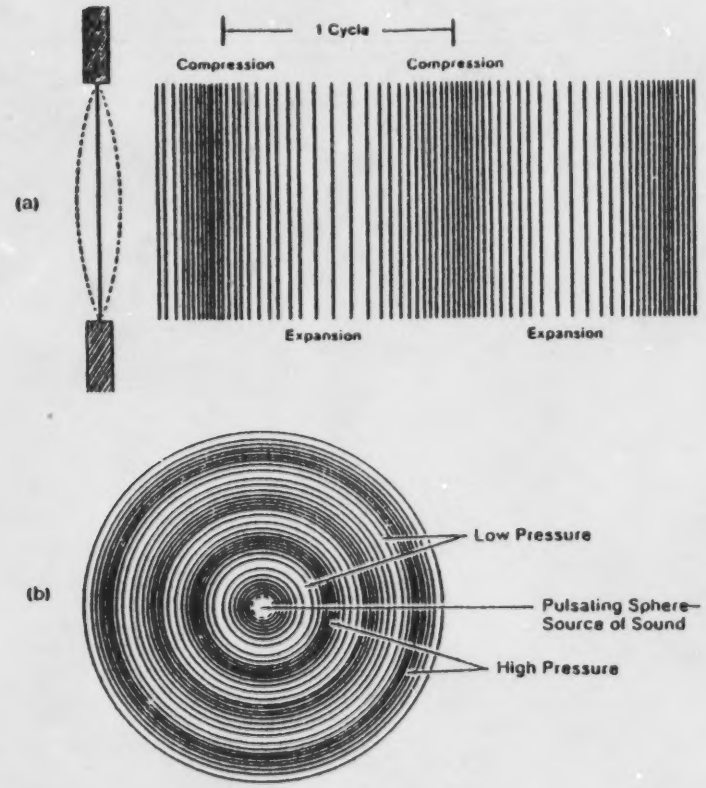


FIG. 3

The compressions (high pressure) are indicated by lines drawn closer together. The rarefactions (low pressure) are indicated by lines drawn further apart.

SOUND PRESSURE MEASUREMENT

Sound pressure, like temperature, is measured relative to some reference point. Because sound pressures are small variations in atmospheric pressure, it is convenient to measure sound pressure in the same unit. Atmospheric pressure is measured in various units:

1 atmosphere = 760 mm Hg = 760 Torr
= 14.7 lb/in^2
= 1000 Millibars
= 10^6 dynes/cm^2
= 10^5 Newtons/m^2 = 10^5 pascals.

Measured sound pressures may have little meaning in themselves. When they are related to common sound levels (a whisper, for example), these numbers can be better appreciated. The weakest sound pressure that can be perceived as sound is a very small quantity; however, the range of sound pressure perceived as sound by the human ear is extremely large. The zero level of sound pressure, for example, is not a true zero in measured pressure. The zero level of sound pressure is defined as the average human threshold of audibility at about 1000 Hz. Sound pressures audible to the human ear range between the threshold of hearing (0.00002 N/m^2) and the threshold of feeling or pain (20 N/m^2). This range corresponds to a ratio of 1 to a million.

Figure 4 relates sound pressure in N/m^2 with frequently-experienced sound levels. Also shown are sound pressure levels as measured in decibels, defined in the following paragraphs.

Selection of a practical scale for sound measurements involves two problems. The first is concerned with spanning the tremendous range of sound pressures encountered; the second is concerned with the nonlinear manner in which the ear responds. The ear

FIG. 4

tends to respond logarithmically to the intensity of an acoustic stimulus. Both of these problems can be solved by the use of a logarithmic or a decibel scale. One advantage of the decibel scale is that it can represent, on ordinary graph paper, a large range of sound pressures, so that large variations are as easily represented as small ones.

The decibel is a dimensionless unit used to express the logarithm of the ratio of two sound intensities. Accordingly, the intensity level of a sound wave is defined by Equation 1.

$$I = 10 \log \frac{p^2}{p_o^2} \qquad (1)$$

where

I = intensity level of sound wave in decibels (dB).
p^2 = sound pressure produced in N/m^2.
p_o = reference pressure, 0.00002 N/m^2 (corresponding to the faintest sound signal that can be heard).

The pressures are squared because the sound intensity (energy) of the sound wave is proportional to the square of the pressure. Use of this equation is illustrated in Example A.

EXAMPLE A: INTENSITY LEVEL OF SOUND WAVES

Given: An air conditioner produces sound pressures of 0.2 N/m^2.

Find: Intensity level in decibel notation.

Solution:

$$I = 10 \log \frac{p^2}{p_o^2}$$

$$= 10 \log \frac{0.02^2}{0.00002}$$

$$= 10 \log 10^6$$

$$= 10 \times 6$$

$$I = 60 \text{ dB}$$

On the decibel scale, this sound pressure is equal to 60 dB, measured relative to 0.00002 N/m.

Because sound levels in decibels involve logarithms of intensity ratios, they are not proportional to intensities, and, consequently, cannot be added or subtracted in the simple way that intensities can be. If, for example, the intensity of a sound is doubled, the sound level in decibels is not doubled. Instead, Equation 1 shows that there is an increase of only 3 dB in the sound level. If one machine produces a sound of 90 dB, and a second identical machine is placed adjacent to the first, the combined sound would be 93 dB, not 180 dB.

On the decibel scale, zero is the threshold of hearing, and 120 dB is the threshold of pain. As explained, the reference base for the decibel was chosen as the lower limit of hearing, or the threshold of audibility, for a person with good hearing. This threshold (or reference base) is 0.00002 Newton/m^2 or, by Equation 1, 0 dB. On the other hand, the maximum sound that the ear can sense is about 4 lb/sq ft, or 200 N/m^2 — actually about 1/4 of atmospheric pressure, or 140 on the decibel scale. Thus, sound levels from 0 dB to 140 dB range from the threshold of hearing, through the threshold of pain, to the maximum response of the ear.

Various sound levels in dB are related to common sounds in Figure 5. The gentle rustle of leaves, for example, is rated at 20 decibels, and a typical office has a 50-decibel background noise. Moderate traffic noise ranges around 70 decibels, and a police whistle reaches 80. Subways and elevated trains rank just below thunder at 100 decibels. At just about 120 decibels, the ear begins to feel pain. Sound levels usually are measured at their source; therefore, decibel ratings decrease as the distance from the source increases. These ratings, thus, should be regarded as averages and should be used primarily for comparative purposes.

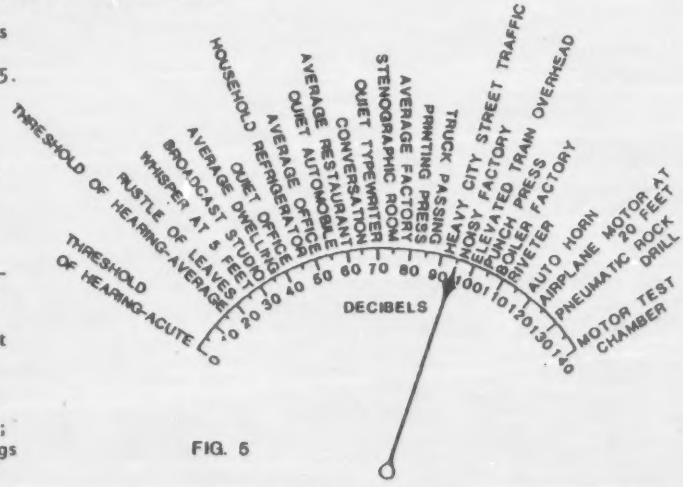


FIG. 5

SUMMARY

Although sound cannot travel through a vacuum, it does travel through air, other gases, liquids, and solids. Sound waves are small variations in pressure superimposed upon the normal atmospheric pressure by a vibrating source and transmitted as a longitudinal-pressure wave motion. Devices that measure sound actually measure the sound pressure (the increased pressure caused by the waves). Any given intensity of sound is accompanied by a corresponding sound pressure.

MEASUREMENTS OF TYPICAL SOUND PRESSURE LEVELS

MATERIALS

Sound level meter — General Radio model GR 1565-B.
Band saw, vacuum pump, drill press, compressor, or other noise source in the laboratory.

PROCEDURE

1. Take 5 to 10 minutes to learn how to make accurate measurements with sound level meter. Read instructions, and have your instructor evaluate your understanding of the use of this instrument.

2. Push the weighting-control button for the "A" scale.

3. Set meter response mode to "slow." This setting will tend to average out highly-fluctuating noise and to make the meter easier to read.

4. You now are ready to make a measurement.

5. In your laboratory area, point the microphone in the direction of noise source.

6. Turn range-control knob until meter reads on scale, and read sound pressure level in decibels, directly from the sum of range control setting and meter reading. For instance, if the range control is set at 70 to 80 and the meter reads +3, the sound pressure is 73 decibels.

7. Record your measurement in Data Table.

8. Make additional sound-level measurements in a noisy area, such as in a machine shop, in the street, in a classroom, or in a library.

9. Record your measurements in Data Table.

10. Discuss some practical uses of sound measurement. (Some hints are detection, reduction of sound, and shop layout.)

DATA TABLE	
LOCATION	SOUND LEVEL IN dB
Laboratory	
Machine Shop	
Street	
Classroom	
Library	

CONTOURS OF EQUAL SOUND-PRESSURE LEVELS AROUND A NOISE SOURCE

MATERIALS

Same as for first experiment.

PROCEDURE

1. In an area relatively free of obstructions and walls, turn on a piece of equipment such as a vacuum pump, a drill press, or a compressor.

2. As in the first experiment, use the sound level meter to determine the location of the highest sound-pressure level produced by this equipment. This point obviously will be very close to the equipment.

3. If the highest sound-pressure level produced by this equipment was, say 83 dB, set the sound level meter to a lower value, say, 70 dB.

4. Walk around the noisy equipment, making measurements every few feet. Be sure to move in or out as you go around the equipment in order to ensure that the meter always reads 70 dB.

5. Mark this 70-dB contour on the floor around the equipment.

6. Repeat steps 3-5, setting the sound level meter first at 65 dB and then at 60 dB, and determine sound pressure contours for each setting.

7. When you finish, you should have contours of equal sound pressure around your noise source, as shown in Figure 6.

8. Make a scale drawing of your results.

9. If the contours are uneven, why?

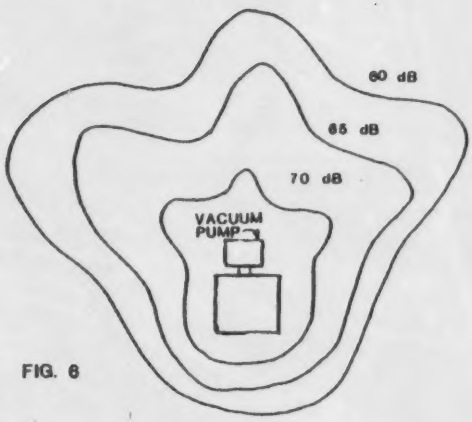


FIG. 6

SELF CHECK

1. Draw and label a sketch of a plane and spherical sound wave.

2. List advantages of the decibel notation.

3. Use Equation 1 to show directly that the intensity level for a sound signal of 0.00002 N/m^2 is 0 dB.

4. A lawn mower produces a sound pressure of approximately 1 N/m^2. Determine its intensity level in dB.

5. If three lawn mowers identical to the one of Problem 4 are operated side-by-side, what is their combined sound pressure level in N/m^2 and intensity level in dB?

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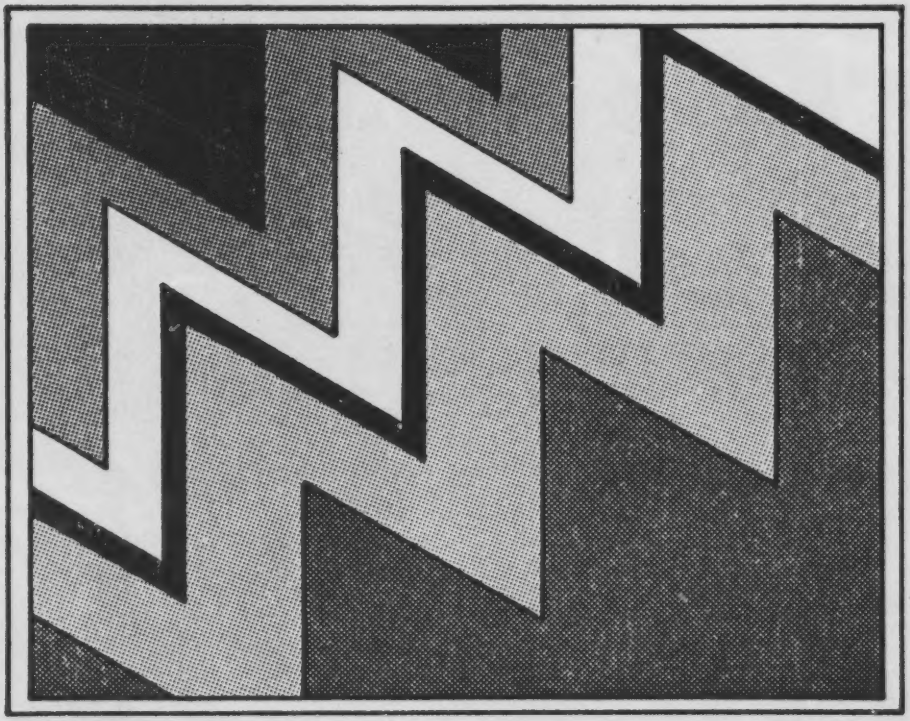
UNIFIED TECHNICAL CONCEPTS

MODULE 11 E 1

ALTERNATING VOLTAGE AND CURRENT



ELECTRICAL APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

When a voltage periodically changes with respect to time, it is referred to as an alternating voltage. This alternating, wave-like voltage, when applied to a complete circuit, will cause an alternating current to flow. Several devices used in electronic circuits react differently to alternating current than they do to direct current. This module compares and contrasts a.c. and d.c. characteristics and describes the effects they produce in various circuit components, such as resistors, capacitors, and coils. In the laboratory, the student will measure peak-to-peak voltage and will compare resistance, inductive reactance, and capactive reactance.

PREREQUISITES

Student should have completed Concept Module II-O, "Vibrations and Waves."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following in one or two sentences:

  Frequency of alternating voltages.
  Peak-to-peak voltages.
  Rms current and voltage.
  Instantaneous voltages.
  Inductive reactances.
  Capacitive reactances.
  Effective voltages.

- List two electrical components that respond differently and one that responds the same to a.c. and d.c.

- Given a capacitor and/or coil, calculate capacitive or inductive reactance. Use Ohm's law to determine value of the current flow at a specific voltage.

- Given the appropriate equipment, measure peak-to-peak voltage; and compare resistance, inductance, and capacitive reactance.

DISCUSSION

The use of a graph is often a convenient means of visualizing physical qualities such as velocity, power, rate, etc. Graphical visualization also is useful in visualizing direct and alternating voltages. A graph of a direct voltage versus time is shown in Figure 1, in which a straight line parallel to the time axis indicates that the voltage remains constant over a time period.

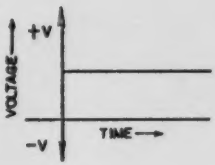


FIG. 1

Conversely, an alternating voltage changes with respect to time, as indicated by the term "alternating." The graph of voltage (Figure 2) follows the form of a trigonometric sine function. Mathematically this graph follows Equation 1.

$$V = V_{max} \sin (2\pi ft) \qquad (1)$$

where

V = voltage at any instant of time t.
V_{max} = voltage at point b in Figure 2.
f = frequency in hertz.
t = time in seconds.

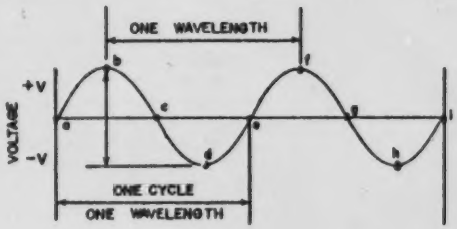


FIG. 2

Over a period of time (defined as one period), alternating voltage repeats itself. Figure 2 shows two and one-half cycles of the voltage change. Concept Module II-0 gives definitions of wavelength, frequency, and period. In Figure 2, the distance from points a to e, from b to f, or, in fact, between any two corresponding points, is the wavelength. In electrical applications, the more common term is "frequency" — the number of times the wave repeats itself each second. The "hertz" unit is used to represent one complete cycle each second, as shown by Equation 2.

$$1 \text{ Hz} = 1 \frac{\text{cycle}}{\text{s}} \qquad (2)$$

Commercially-produced electricity in the United States is alternating at a frequency of 60 hertz, although many other frequencies are used in various devices, such as radios, television sets, etc.

EFFECTIVE VOLTAGE

If the value of voltage is always changing, how can one value (such as 120 V) be applied to alternating voltage? The obvious answer to this question would be an "average" value; however, a simple average would be useless, because there are as many negative values of voltage as there are positive values.

A new term, "peak-to-peak voltage," is applied to alternating voltage waves. This very descriptive term defines the voltage measured from the peak of the crest to the bottom of the trough. Peak-to-peak voltage is shown in Figure 2 as the vertical measurement from points b to d. Notice that this value is twice as great as the wave amplitude defined in Module 11-0. As useful as the peak-to-peak value of voltage may be, it does not represent a useful comparison to direct voltage when various effects are measured.

Recall from previous modules using direct current that a direct voltage applied across a given resistance will produce a fixed amount of heat. This heating effect can be used to specify one value for an alternating voltage. A definition therefore could be stated in the following way: one volt a.c. is the alternating voltage that will produce the same amount of heat in a given resistor as one volt d.c. This value is called the "effective voltage." It is the voltage recorded by most voltmeters. Effective voltage is often called rms voltage. The abbreviation rms stands for root mean square, which is a mathematical averaging technique that does not need to be learned here. When the root mean square technique is applied to an alternating voltage sine wave, the rms voltage is found to have the value indicated by Equation 3.

$$V_{rms} = 0.707\ V_{max} \qquad (3)$$

where

V_{rms} = effective voltage.

An alternating voltage, therefore, that has an effective voltage (rms) of 1 volt must have a maximum voltage at the crest of 1.414 volts and a peak-to-peak voltage of 2.83 volts. Standard household voltage of 115 effective volts will have a maximum voltage (amplitude) of 162.5 volts and a peak-to-peak value of 325 volts.

The internal heating effect produced in a resistor is the same, therefore, for one volt d.c. as for one rms volt a.c. On the other hand, the inductance coil and the capacitor behave differently when connected to an alternating voltage than when connected to direct voltage. The reaction of these two elements to an alternating voltage can be demonstrated with a simple resistor such as a long, straight, high-resistance wire. When a direct

voltage is applied to this wire, a current is produced constant and proportional to the voltage applied, as described by Ohm's law. An alternating voltage applied to the same wire also will cause a proportional current to flow, but the current will be alternating in step with the voltage. In other words, when the voltage is greatest in one polarity, the current flow will be greatest also; but when the voltage reverses polarity (becomes negative in Figure 2), the current flow will reverse direction as well. This is illustrated graphically in Figure 3. The resistance of the wire is the same regardless of its connection to direct or alternating voltage.

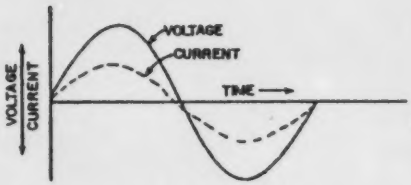


FIG. 3

INDUCTIVE REACTANCE

A coil, on the other hand, reacts against an applied voltage. When voltage is applied to a coil, the current produced establishes a magnetic field around the coil similar to the magnetic field around a bar magnet (Figure 4).

FIG. 4

Direct current through the coil establishes a constant and stationary magnetic field. Because the strength of the field depends upon the amount of current through the coil, an alternating current will produce a continually-changing magnetic field. As the current alternates, the magnetic lines of force first expand outward, then reach a maximum extent, followed by a collapse as the current reverses direction. The continually-moving magnetic field near the conducting coil will cause an emf to be induced in the coil according to Lenz's law. The induced voltage is called a counter emf, because it is opposite in polarity to the initially-applied voltage. The counter emf opposes the current flow that produced it. A coil, therefore, opposes the flow of alternating current in a manner different from the way in which it opposes direct current. The extra opposition to alternating current flow is called "inductive reactance." It is measured in ohms and depends upon frequency of the applied voltage and upon construction of the coil (called "inductance"), related in Equation 4.

$$X_L = 2\pi fL \qquad (4)$$

where

X_L = inductive reactances in ohms.
f = frequency of a.c. in hertz.
L = inductance of coil in henries.

This relationship is clarified further by Examples A and B, in which Equation 4 is used to solve problems.

EXAMPLE A: CALCULATION OF INDUCTIVE REACTANCE	
Given:	A 250-millihenry coil is placed in a 60-hertz a.c. circuit.
Find:	The inductive reactance.
Solution:	(From Equation 4.) $X_L = 2\pi fL$ $= 2\pi\ (60\ Hz)(250 \times 10^{-3}\ henry)$ $X_L = 94\ ohms$

EXAMPLE B: INDUCTIVE REACTANCE IN HIGH-FREQUENCY CIRCUIT	
Given:	A 250-millihenry coil is placed in a 3000-hertz a.c. circuit.
Find:	The inductive reactance.
Solution:	(From Equation 4.) $X_L = 2\pi fL$ $= 2\pi\ (3000\ Hz)(250 \times 10^{-3}\ henry)$ $X_L = 4710\ ohms$

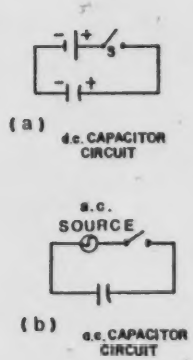


FIG. 5

CAPACITIVE REACTANCE

A capacitor also behaves differently in an alternating-current circuit than in a direct-current circuit. When the switch in the d.c. circuit shown in Figure 5a is closed, the capacitor simply charges to the voltage of the source and then prevents additional current flow, because the capacitor voltage opposes the source voltage. A capacitor, therefore, acts as an infinite resistance (open circuit) to direct current.

In the a.c. circuit shown in Figure 5b, the capacitor first charges to a maximum, then discharges as the voltage of the source decreases. When the source voltage reverses, the discharging capacitor actually assists the source, which causes the capacitor to offer very little opposition to rapidly-changing current. The opposition of a capacitor, called "capacitive reactance," is given by Equation 5:

$$X_C = \frac{1}{2\pi fC} \qquad (5)$$

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where

X_C = capacitive reactance in ohms.
f = frequency of a.c. in henries.
C = capacitance of capacitor in farads.

Equation 5 is clarified further by Examples C and D.

Both coils and capacitors produce opposition to alternating current that they do not cause in direct-current circuits. The equations presented in this module indicate that the reactances, sometimes called "a.c. resistance," depend upon frequency of the alternating current, as well as upon design of the coil or capacitor.

EXAMPLE C: CALCULATION OF CAPACITANCE

Given: A 0.2-μF capacitor connected to a 60-hertz a.c. circuit.

Find: Capacitive reactance.

Solution: (From Equation 5.)

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1}{2\pi\ (60\ \text{Hz})(0.2 \times 10^{-6}\ \text{farads})}$$

$$X_C = 13{,}300\ \text{ohms}$$

EXAMPLE D: CAPACITANCE IN A HIGH-FREQUENCY CIRCUIT

Given: A 0.2-μF capacitor placed in a 10,000-hertz a.c. circuit.

Find: What capacitive reactance would be caused.

Solution: (From Equation 5.)

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1}{2\pi\ (10 \times 10^{3}\ \text{Hz})(0.2 \times 10^{-6}\ \text{F})}$$

$$X_C = 80\ \text{ohms}$$

SUMMARY

Alternating currents and voltages exhibit a wave-like, or alternating, quality. Frequency is the number of complete cycles within a given time (1 Hz = 1 cycle per second). One volt a.c. is the amount of voltage that will produce the same amount of heat in a given resistor as one volt d.c. A peak-to-peak voltage of 2.83 V is required to equal one d.c. volt. The behaviour of d.c. and a.c. voltages differs in circuits having inductive reactance or capacitive reactance.

MEASURING A.C. VOLTAGE

MATERIALS

5-V dry cell.
100-Ω, 1/2 watt resistor.
Switch (single pole, single throw).
Oscilloscope (student type).
a.c. signal generator.
Vacuum tube voltmeter (VTVM).
Ammeter.
2-μF capacitor.
250-μH coil

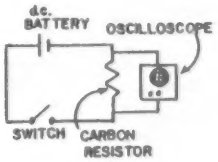

FIG. 6

PROCEDURE

In this experiment an oscilloscope is used. A complete understanding of operation of the oscilloscope is unnecessary at this time. Student should view the instrument as a sophisticated voltmeter that electronically produces a visual pattern of the voltage. With aid of the instructor, the student will calibrate an oscilloscope to determine correct voltage readings.

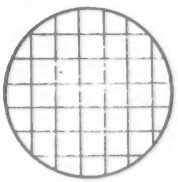

FIG. 7

This experiment will help the student to visualize the difference between d.c. and a.c. signals and will illustrate graphically the rms and peak-to-peak alternating voltages.

1. Connect circuit as shown in Figure 6. Observe and draw oscilloscope pattern on Figure 7 when switch is closed.

2. Replace d.c. battery with a low-voltage a.c. signal generator (Figure 8). Adjust signal generator for 60 hertz and then for 10,000 hertz; draw oscilloscope patterns on Figure 9.

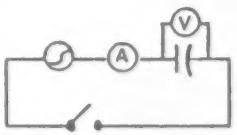

FIG. 8

3. With signal generator connected as in Figure 8, measure peak-to-peak voltage with the oscilloscope; use Equation 3 to determine rms voltage. Without changing signal generator, replace oscilloscope with a VTVM; measure rms voltage directly to verify calculated value.

4. What is peak-to-peak voltage?

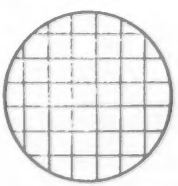

FIG. 9

RESISTANCE, INDUCTIVE REACTANCE, AND CAPACITIVE REACTANCE

MATERIALS

5-V dry cell.
100-Ω, 1/2-watt resistor.
Switch (single pole, single throw).
Oscilloscope (student type).
a.c. signal generator.
Vacuum tube voltmeter (VTVM).
Ammeter.
2-μF capacitor.
250-μH coil.

PROCEDURE

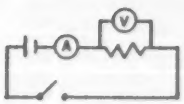

FIG. 10

This experiment compares resistance, inductive reactances and capacitive reactances.

1. Arrange d.c. circuit shown in Figure 10. Measure appropriate current and voltage; then calculate resistance of resistor with ohm's law.

2. As in the first experiment, replace d.c. battery with a low-voltage a.c. signal generator (Figure 8). Switch meter selector switches from d.c. to a.c. Again, determine resistance by using Ohm's law; compare with value determined in d.c. circuit.

3. Use Ohm's law to determine opposition to current flow caused by a capacitor. Use circuit shown in Figure 8. Adjust signal generator to 60 Hz; find capacitive reactance by equation:

$$X_C = \frac{V}{I} \qquad (6)$$

where
V = a.c. voltage read on voltmeter.
I = a.c. current read on ammeter.

4. Repeat number 3 with signal generator set at 10,000 Hz.

5. Use Equation 5 to compare values obtained for X_C.

6. Repeat steps 1-5 with a coil in circuit.

7. Explain the difference between the qualities measured.

SELF CHECK

1. Define in one or two sentences the following:

   Frequency of alternating voltages.
   Peak-to-peak voltages.
   Rms current and voltage.
   Instantaneous voltages.
   Inductive reactances.
   Capacitive reactances.
   Effective voltages.

2. The peak-to-peak voltage measured with an oscilloscope is 150 volts; what corresponding rms value should be recorded by a voltmeter?

3. List two electrical components that respond differently and one that responds the same to a.c. and d.c.

4. Calculate the capacitive reactance in ohms of a 2-μF capacitor at 5000 Hz.

5. If a 1-μF capacitor is connected to a 117-volt, 60-hertz line, how much current will flow?

6. A 1-henry coil is placed in a 117-volt, 60-hertz line; what is the inductive reactance and how much current will flow?

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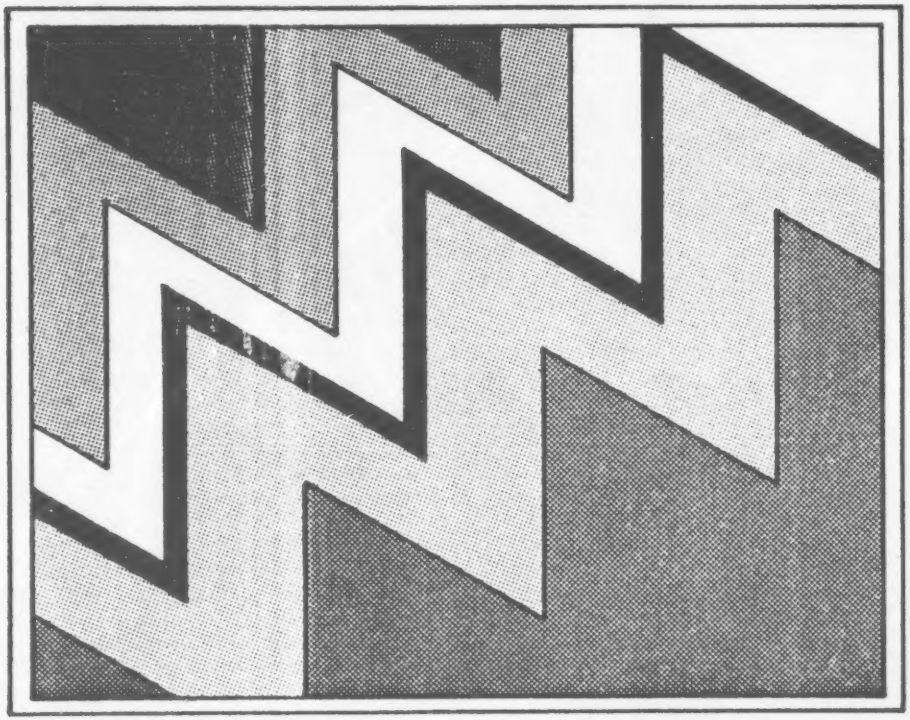
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 11 E 2

PHASE RELATIONSHIPS IN AC CIRCUITS



ELECTRICAL APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center – Southwest

TERC – SW

INTRODUCTION

PHASE RELATIONS

Direct current encounters opposition from the resistance of conductors. Alternating (wave-like) current encounters the same resistive opposition; but in addition, alternating current experiences opposition in the form of capacitive and inductive reactances. These reactances cause the current and voltage to be out of step with each other. Out-of-step problems cause loss of power in alternating-current circuits.

This module investigates the details of capacitive and inductive reactances, their measurements, and how they affect alternating-current circuits. In the laboratory, the student will determine the impedance, phase angle, and power factor of RC, RL, and RLC circuits.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Draw sine wave sketches, showing the phase relationships between current and voltage in R (purely resistive), RL (resistive-inductive), RC (resistive-capacitive), and RLC (resistive-inductive-capacitive) series circuits.
- Draw and interpret vector diagrams of the phase relationship in RC, RL, and RLC circuits; and determine phase angle and impedance.
- Define these terms:

  Impedance.
  True power.
  Apparent power.

Power factor.
Phase angle.
Resonant circuit.

- Given a series a.c. circuit, calculate the value of additional circuit components necessary to produce resonance (power factor of one).

- Given the appropriate equipment, determine the impedance, phase angle, and power factor from current and voltage measurements taken in RLC, RL, and RC series circuits.

PHASE RELATIONS IN RESISTIVE CIRCUITS

In a.c. circuits that contain only resistance, the current flow is in phase with the voltage. The current responds immediately to the voltage, as shown graphically in Figure 1. The power at any instant is the algebraic product of the voltage and current at that instant (Equation 1). Power varies according to the graph in Figure 2.

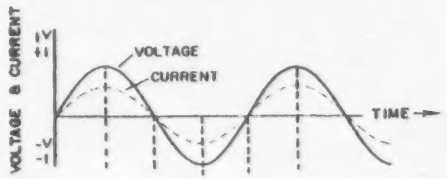


FIG. 1

$$P = VI \qquad (1)$$

where

P = power.
V = voltage.
I = current.

Notice, also, the value of "average power" given in Figure 2. The average power is the value of the constant power that would produce the same effect as the varying power. Notice that when the current and voltage are in phase, the power always is a positive value.

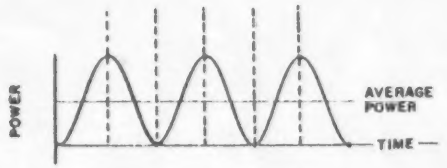


FIG. 2

PHASE RELATIONS IN INDUCTIVE CIRCUITS

When current through a coil varies, as it must in an a.c. circuit, it produces a magnetic field that expands as the current increases and collapses as it decreases. This expanding and collapsing magnetic field, in turn, causes a counter voltage, which opposes change in the flow of current, to be induced in the coil. The additional opposition to current flow is called "inductive reactance" and is expressed in Equation 2:

$$X_L = 2\pi fL \qquad (2)$$

where

X_L = inductive reactance in ohms.
f = frequency of a.c. in Hz.
L = inductance of coil in henries.

Not only does the coil resist a changing current, but it does so in such a way as to cause the current and voltage to be out of phase; that is, out of step in time. In a purely inductive circuit (one containing only a coil with no resistance), the current will lag behind the voltage by one-fourth a period, or 90° (π/2 radians), as shown graphically in Figure 3. Figure 4, a graph of power versus time, shows the power dissipated by the inductance. The power is positive when both the current and voltage are positive or when they are both negative, but the power is negative when current and voltage are of opposite polarity. The variation of power shown in Figure 4 leads to a zero-average power dissipation by the coil.

FIG. 3

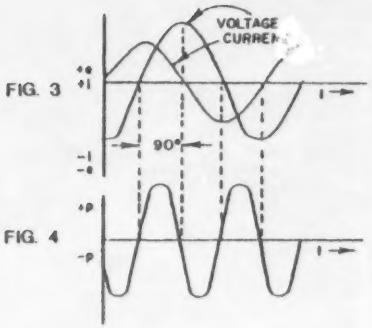


FIG. 4

PHASE RELATIONS IN RL CIRCUITS

A RL circuit is one that contains both resistance and inductance. Since a coil made of wire cannot be free of all resistance, a RL circuit is a more realistic circuit than is the purely inductive circuit represented in Figures 3 and 4.

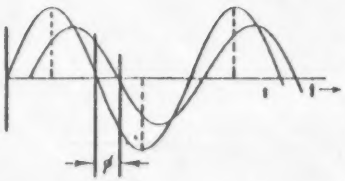


FIG. 5

A circuit containing both resistance and inductance will develop a phase angle difference (ϕ) of between 0° and 90° between voltage and current (Figure 5).

A RL circuit is shown schematically in Figure 6. To determine the total opposition to current, since this is a series circuit, one might expect to add X_L and R. This, however, is incorrect, because of the phase difference between the current and voltage. A procedure identical to that of adding vectors can be implemented. This procedure will give the correct total opposition to alternating current, called "impedance," and the correct phase angle between voltage variation and current variation.

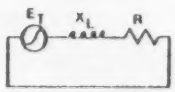


FIG. 6

The procedure is as follows:

(1) On an x-y graph, plot the value of resistance R (in ohms) along the positive x-axis, according to an appropriate scale. This plot will be the horizontal vector R.

(2) Use the parallelogram method to add X_L and R. The resultant will be the total opposition (impedance), and the angle between the impedance vector and the resistance vector is the phase angle.

This procedure is demonstrated in Example A.

EXAMPLE A: PHASE ANGLE AND IMPEDANCE OF RL CIRCUIT

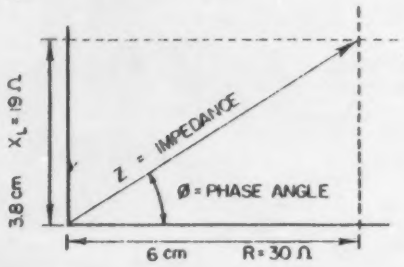
Given: A 1500-Hz (hertz) a.c. circuit contains a 30Ω (ohm) resistor and a 2 mH (millihenry) coil.

Find: Phase angle and impedance.

Solution: $X_L = 2\pi fL$

$= 2\pi\ (1500\ Hz)(2 \times 10^{-3}\ H)$

$X_L = 19\Omega$



SCALE: 1 cm = 5Ω

From the vector diagram, length of the impedance vector is 7.1 cm; therefore, 7.1 cm = 5Ω/cm = 35.5Ω. Also from vector diagram —

$$\tan \phi = \frac{X_L}{R}$$

$$= \frac{19}{30}$$

$$\tan \phi = 0.633$$

$$\therefore \phi = 32.3°$$

Thus, current lags voltage by 32.3°.

POWER IN A RL CIRCUIT

When a voltmeter and ammeter are connected in an a.c. circuit, they record the rms values of voltage and current. If these values are multiplied together, one apparently would find the power dissipated in the circuit. The product of rms voltage and current gives what is called "apparent" power. If the voltage and current were in phase, this product (apparent power) also would be the power dissipated by the circuit. But since the voltage and current are

out of phase by an amount equal to the phase angle, the apparent power is greater than the true power. The true power can be determined by Equation 3 in an a.c. circuit when the phase angle is known.

$$P_T = V_{rms}\ I_{rms} \cos \phi \qquad (3)$$

where

P_T = true power.
ϕ = phase angle (power factor).
$V_{rms}\ I_{rms}$ = apparent power.

True power in a RL circuit is illustrated graphically in Figure 7.

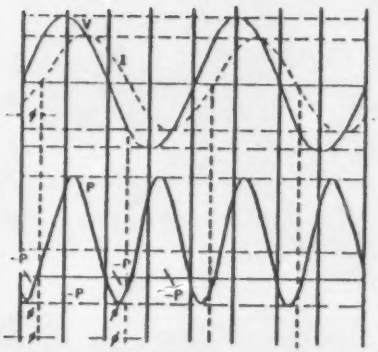


FIG. 7

Notice that when both current and voltage are positive or negative, a positive value for power results; but when either current or voltage is negative, and the other one positive, a negative power results. In Figure 7, more positive power is present than negative power; thus, the average power expended is positive as in a pure resistance. Compare the power graph in Figure 7 with the graphs in Figures 2 and 4. From these graphs and from Equation 3, the true useful power obviously can vary from zero in a pure inductive circuit to a maximum amount in a pure resistive circuit as the phase angle changes from 90° to 0°.

The term "cos ϕ" in Equation 3 is called the "power factor." It relates the true and apparent power in a given circuit. This relation can be seen when Equation 3 is rearranged to become Equation 4.

$$\cos \phi = \frac{P_T}{P_A} \qquad (4)$$

where

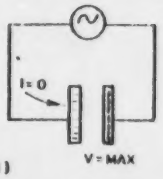
P_A = apparent power.

True power is often measured in units of watts; whereas apparent power is expressed in units of volt-amps.

PHASE RELATIONS IN CAPACITIVE CIRCUITS

When a capacitor is connected in an a.c. circuit, the opposition presented to the alternating current is called "capacitive reactance," the value of which can be determined from Equation 5.

$$X_C = \frac{1}{2\pi fC} \qquad (5)$$



(a)

where

X_C = capacitive reactance.
π = pi, 3.14.
C = capacitance.

When an alternating voltage is applied to a capacitor, current flows easily to the capacitor plates until enough charge is collected on the plates to build up a significant capacitor voltage. Then, as the capacitor voltage is large, it prevents additional current from flowing to the capacitor plates (Figure 8).

(b)

FIG. 8

Graphically, the phase relations in a capacitive circuit are illustrated in Figure 9. In this case, the current is said to "lead" the voltage by 90°. (Compare with Figure 4.)

An analysis similar to that for power in inductive circuits can be used for capacitor circuits. The power graph in Figure 10 is the result: the average power dissipated by a capacitor in an a.c. circuit is zero. This relationship follows from Equation 3.

FIG. 9

$$P_T = V_{rms}\ I_{rms} \cos \phi \qquad (3)$$

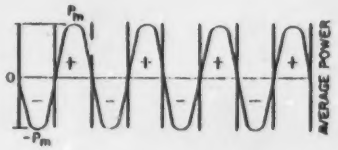


FIG. 10

where

cos ϕ = phase angle (90° in a capacitive circuit).
$P_T = V_{rms}\ I_{rms} \cos 90°$.
$P_T = V_{rms}\ I_{rms}\ (0)$.
$P_T = 0$.

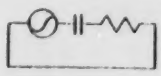
As a consequence of the above values, neither pure inductances nor pure capacitances dissipate any power.

PHASE RELATIONS IN RESISTANCE AND CAPACITANCE CIRCUITS

The phase angle in an a.c. circuit containing both resistance and capacitance will be between 0 and 90°. Vector diagrams can be drawn to determine precisely the impedance and the phase angle; the procedure is similar to that outlined for inductive reactance in Example A. The only modification in the procedure is this: the capacitive reactances must be plotted down on the negative y axis to account for the fact that current leads by 90° in a capacitive circuit, but lags by 90° in an inductive circuit. The inductive reactances and capacitive reactances, therefore, are plotted 180° from each other. (Refer to Example B, which consists of an RC circuit problem and solution.)

EXAMPLE B: PHASE ANGLE AND IMPEDANCE OF A RC CIRCUIT

Given: A series circuit consists of a 1000Ω resistor and a 1 μF (microfarad) capacitor connected to a 117-V (volt) 60-Hz source.



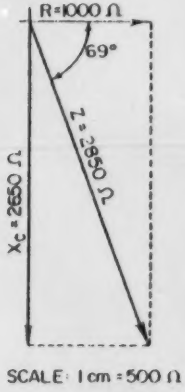
Find: Phase angle and total impedance of circuit.

Solution:

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1}{2\pi\ (60)\ (1 \times 10^{-6} f)}$$

$$X_C = 2650\Omega$$



SCALE: 1cm = 500 Ω

Verify the result mathematically:

$$1000^2 + 2650^2 = z^2$$

$$z = 2832\Omega$$

The phase angle is determined:

$$\tan \phi = \frac{X_C}{R}$$

$$= \frac{2650}{1000}$$

$$= 2.65$$

$$\tan \phi = 69°$$

Determine both the true and the apparent power in the circuit:

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z}$$

$$I = \frac{117V}{2832\Omega}$$

$$I = 0.04 \text{ A (amps)}$$

$$P_A = \mathcal{E}_{rms} I_{rms}$$

$$= (117 \text{ V})(0.04 \text{ A})$$

$$P_A = 4.68 \text{ VA}$$

$$P_T = (4.68 \text{ VA})(\cos 69°)$$

$$P_T = 1.68 \text{ watts}$$

PHASE RELATIONS IN SERIES RLC CIRCUITS

Phase relation in a.c. circuits containing capacitive reactances, inductive reactances, and resistance is a simple combination of the concepts presented in the sections on RL, RC, and pure resistance circuits. In fact, the analysis of an RLC circuit will be identical to the analysis of an RL circuit if the inductive reactance exceeds the capacitive reactance. Similarly, an RLC circuit will require the same analysis as a RC circuit if the capacitive reactances are larger than the inductive reactances. If the capacitive and inductive reactances are equal, a special condition called "resonance" exists. A resonant RLC circuit is analyzed in the same way as is purely resistive circuit, as far as phase relations are concerned.

Consider a series RLC circuit with capacitive reactance of X_C ohms, and inductive reactance of X_L ohms, and a resistance of R ohms. Determine the impedance and phase angle in the circuit.

. The following equations are developed for total reactance by use of basic trigonometric and algebraic expressions:

$$X_T = X_L - X_C \tag{6}$$

$$z = \sqrt{R^2 + (X_L - X_C)^2} \tag{7}$$

$$\tan \phi = \frac{X_L - X_C}{R} \tag{8}$$

NOTE: The quantity $X_C - X_L$ simply indicates a difference between the two values. The terms in the equation can be revised, as long as the student analyzes the circuit to determine if it is a capacitive or an inductive circuit.

$$\cos \phi = \frac{R}{z} \qquad (9)$$

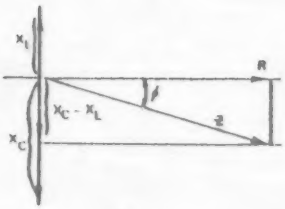


FIG. 11

When X_L and X_C are equal, the value of z will be a maximum equal to R. (See Figure 11.) Since z is minimum at this resonant condition, the current flow will be maximum, and the phase angle equal to zero. At resonance, the relationships are as indicated by Equation 10.

$$X_L = X_C$$

$$2\pi fL = \frac{1}{2\pi fC}$$

$$4\pi^2 f^2 = \frac{1}{LC}$$

$$\text{or} \quad 2\pi f = \frac{1}{\sqrt{LC}} \qquad (10)$$

Besides being point of maximum current flow, a resonant circuit contains the greatest possible power factor, since —

$$\phi = 0$$

$$\cos \phi = 1.$$

When Equation 4 is applied, the apparent power and the true power obviously are equal at resonance. This situation is desirable in commercial power distribution. A power company desires to have the largest possible power factor to provide efficient transmission. Often, a power company will introduce large values of capacitive reactance into a transmission line. This increase compensates for the inductive reactance introduced by large coils in motors or transformers.

Equation 10 also is used in electronic circuits in which the capacitive or inductive reactances are tuned to a desired frequency. This principle is employed in a radio tuning circuit.

SUMMARY

The phase angle ϕ is a measure of how much the voltage leads or lags behind the current in an a.c. circuit. The causes for voltage and current being out of phase stem from the opposition of capacitive and inductive reactances. Loss of power results.

In purely resistive circuits, current is in phase with voltage. Inductive circuits (containing a coil) exhibit "inductive reactance," or opposition to current, which causes the current to lag behind

the voltage by 90°. Circuits containing both resistance and inductance exhibit a phase angle of between 0° and 90°, and the "true power" is less than the expected power ("apparent power") of rms voltage multiplied by current. Capacitive reactance causes current to lead voltage by 90°. In resistance/capacitance circuits, the phase angle is between 0° and 90°. Circuits containing resistance, inductance, and capacitance are in "resonance" if the capacitive and inductive reactances are equal. If the inductive reactance exceeds capacitive reactance, circuit analysis is identical to that for RL circuits, but will be identical to that for RC circuits if capacitive reactance exceeds.

DETERMINATION OF IMPEDANCE, PHASE ANGLE, AND POWER FACTOR

MATERIALS

Resistor, 100 ohms, 1 watt (not wire bound).
Choke coil, 500 mH.
Capacitor, 10 μF, 20 volt rating.
Two switches, SPST.
One a.c. ammeter, 0-0.1 ampere range.
One VTVM.
One low-voltage, 60-Hz power supply, 0-15 volts; or one variable autotransformer.

PROCEDURE

Set up the following circuit illustrated in Figure 12. With switches s_1 and s_2 open, current must go through both the inductor and the capacitor, thus providing a RLC circuit. When s_1 is closed and s_2 open, a RC circuit is produced; with s_1 open and s_2 closed, a RL circuit is available.

Measurements of current and voltage now will be taken and values plotted on a graph. Inductive reactance, capacitive reactance, and resistance will be determined from slope of the graphs; and values of impedance will be compared with the value determined by using Ohm's law.

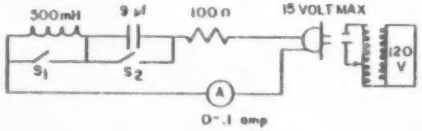


FIG. 12

RLC CIRCUIT

1. With switches s_1 and s_2 open, slowly increase power-supply voltage in uniform increments. Measure voltages across components with VTVM. Record voltage across coil, capacitor, and resistor for each series current reading registered by the ammeter. Complete Data Table 1.

2. Plot voltage versus current for each component on graph in Data Table 1, with current values along horizontal axis and voltage on vertical axis. This plot will produce three lines. Lines may not be perfectly straight because of changes in the components as current increases.

Slope of the graphs will be found from –

$$\text{slope} = \frac{\text{change of voltage}}{\text{change of current}}$$

$$\text{slope} = \frac{\Delta V}{\Delta I}$$

Since $\Delta V/\Delta i$ is the measure of resistance in Ohm's law, the slope of the graph represents opposition to current flow. The following equations can be stated to extend this argument:

$$\frac{\Delta V_L}{\Delta I} = X_L = \text{slope of inductive graph}$$

$$\frac{\Delta V_C}{\Delta I} = X_C = \text{slope of capacitive graph}$$

$$\frac{\Delta V_R}{\Delta I} = R = \text{slope of resistive graph}$$

3. Apply experimental values for X_C, X_L and R in Equation 7 to determine the circuit impedance.

4. Use Equation 8 to determine the phase angle in the circuit.

5. Use Equation 9 to find the power factor of this circuit.

RL CIRCUIT

Close switch s_2 to complete a RL circuit, and perform the same measurements and calculations as in previous procedure on RLC circuit. Use Data Table 2.

RC CIRCUIT

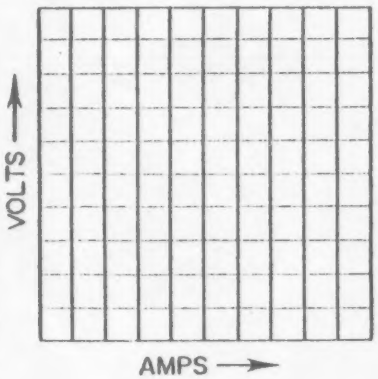
Close switch s_1 and open s_2 to produce a RC, and repeat the same measurements and calculations as in procedure on RLC circuit. Use Data Table 3.

Question for discussion: How can the power factor of an inductive circuit be increased?

DATA TABLE 1: RLC CIRCUIT

RLC CIRCUIT

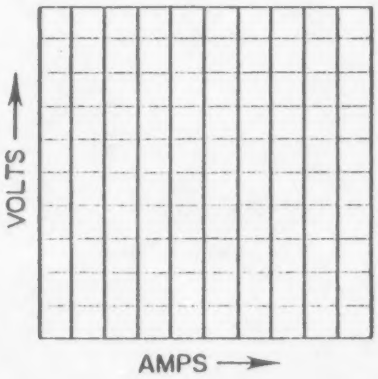
I	V_L	V_C	V_R



DATA TABLE 2: RL CIRCUIT

RL CIRCUIT

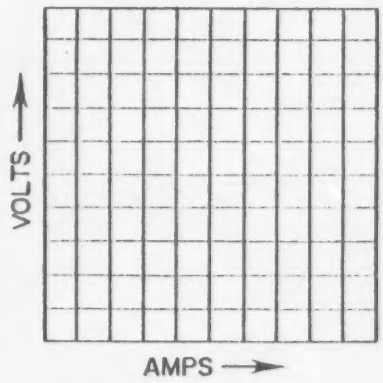
I	V_L	V_R



DATA TABLE 3: RC CIRCUIT

RC CIRCUIT

I	V_C	V_R



SELF CHECK

1. Draw sine wave sketches showing the phase relationship between current and voltage in R, RL, RC, and RLC series circuits. Completely label each sketch, including the phase angle.

2. Define the following terms:

   Impedance.
   True power.
   Apparent power.
   Power factor.
   Phase angle.
   Resonant circuit.

3. Use a vector diagram drawn to scale to determine the impedance and phase angle in a 1500 Hz circuit containing a 300 mH coil and a 700 ohm resistor in series.

4. A 50 millihenry coil is connected to a 60 Hz, 117 V a.c. line containing 80 ohms or resistance. Determine the phase angle, impedance, power factor, true power and apparent power in the circuit.

5. What value of capacitor should be placed in the above circuit to produce a series resonant circuit?

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Harris, Norman C. and Hemmerling, Edwin M. Introductory Applied Physics. 3rd ed. New York: McGraw-Hill Book Company, Inc., 1972.

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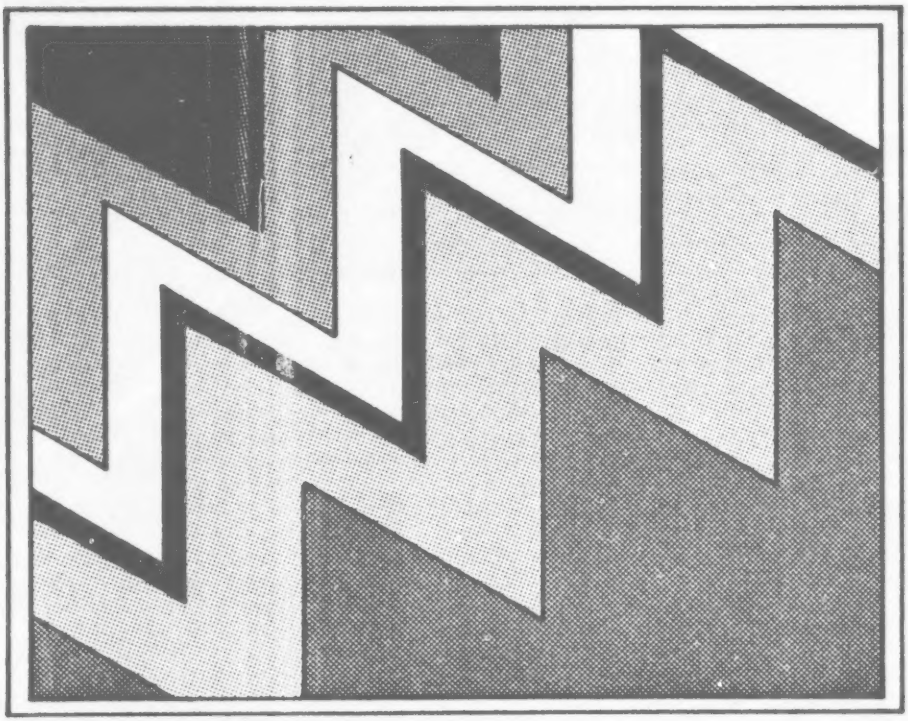
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 11 E 3

OSCILLOSCOPE



ELECTRICAL APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center – Southwest

TERC – SW

INTRODUCTION

This module explains the basics of oscilloscope operation. The oscilloscope is one of the most important electrical devices available for measurement of electrical quantities. It is particularly useful in graphic representations of wave forms that may occur at high frequencies. This module also explains basic operation of a dual-trace oscilloscope, its capability, and its applications. It describes use of Lissajous patterns for frequency determination. In the laboratory, the student will analyze oscilloscope sine waves, phase difference, and Lissajous patterns.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Identify which controls of an oscilloscope operate on the x axis and y axis.
- Describe the use of the z axis control.
- Define time per centimeter.
- Discuss the type of oscilloscope required to measure phase difference.
- List any circuits affected by the oscilloscope trigger control.
- Given the time-control setting and the time required for one cycle, calculate the frequency of the input signal.
- Calculate the voltage displayed on an oscilloscope screen, given the V/cm control setting and the size of the display.

- Describe the precaution necessary in grounding test equipment.

- Given the appropriate equipment, use a dual-trace oscilloscope to determine the following: (1) peak-to-peak and rms values and frequency of a sinusoidal waveshape; (2) frequency of an unknown sinusoidal waveshape (use Lissajous patterns); and (3) phase differences.

CATHODE RAY TUBE CONSTRUCTION

The cathode ray tube (CRT) is the major component of a cathode ray oscilloscope (CRO), known variously as an oscilloscope, O'scope, scope, or oscillograph. The CRT is the component around which all other circuitry in the oscilloscope is designed. The CRT itself is an evacuated tube manufactured in various sizes and shapes, depending upon function. Its general construction is shown (Figure 1).

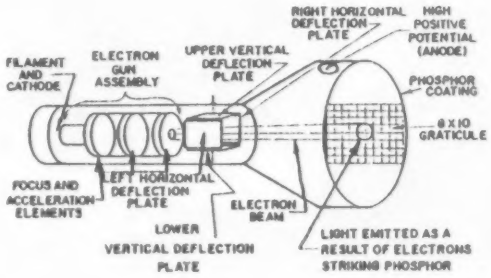


FIG . 1

The electron gun assembly contains the cathode, which emits electrons, and several focusing elements that force the emitted electrons into a beam extending from the electron gun assembly forward to the phosphor screen. The electron gun elements focus both the beam and set beam intensity. Electrons in the beam are accelerated further by a high positive potential placed on the phosphor screen. This voltage may vary from 1000 V to 25 kilovolts, depending on size and function of the CRT. When electrons strike the phosphor, they transfer their kinetic energy to the phosphor, which in turn emits photons of energy (visible light) on the face of the tube.

In the middle of the CRT are deflection elements. The deflection elements are in pairs, with two horizontal plates causing vertical deflection, and two vertical plates causing horizontal deflection of the electron beam. The plates are charged like opposing plates of a capacitor. A positively charged deflection plate pulls the negative stream of electrons toward it, while a negatively charge deflection plate repels the negative stream of electrons from it. Each pair of plates receives across it a voltage, which determines either vertical or horizontal deflection of the beam. Thus, the electron beam is deflected as it passes through crossed electric fields. Since electrons can also be deflected by magnetic fields, the process described here is called electrostatic deflection.

Figure 2 shows various functional blocks of a single-trace oscilloscope with a triggered time base for horizontal deflection. Various operator controls are shown beside their corresponding circuitry.

The vertical-attenuator control limits the input signal to

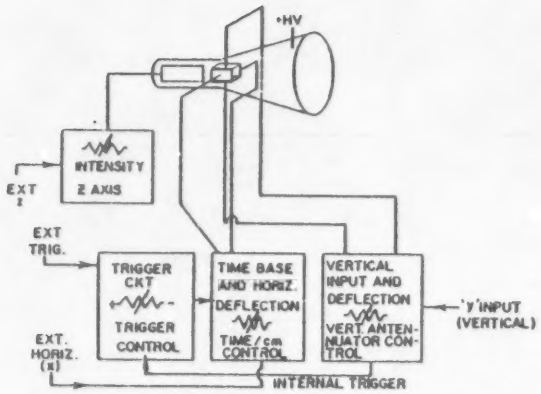


FIG . 2

the vertical deflection circuitry. This determines vertical deflection of the electron beam for a given input voltage. Normally, vertical-attenuator control is provided in steps of 1, 2, and 5. An example of a vertical attenuator control dial is shown (Figure 3). The setting shown (Figure 3) is 50 mV (millivolts)/cm. This value means that for every 50 mV of input signal (the "y" input), the beam will deflect vertically 1 cm on the phosphor face of the tube.

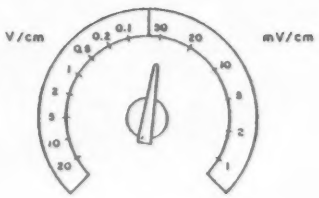


FIG . 3

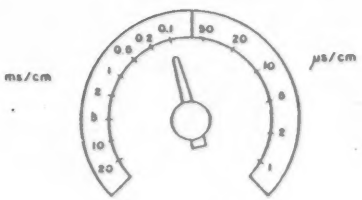


FIG . 4

The horizontal control dial for the time base is essentially the same as the vertical attenuator, except that it controls the x axis and is normally in units of time (Figure 4). The setting shown (Figure 4) is 0.1 ms (millisecond)/cm, which means the trace will cover one centimeter in a horizontal direction in 0.1 ms. If the graticule contains 10 cm (centimeters) horizontally, the trace will cover the complete screen in (10 x 0.1) or 1 ms. The x/y input position allows the operator the choice of inserting his own x-axis input. In different applications, this voltage might represent a time or a frequency.

The trigger input circuit determines starting point of the trace on the "y" input signal. It can also determine level of "y" input signal necessary to cause a trace to be triggered. Most oscilloscopes have an automatic trace feature that will allow a trace to appear in the absence of any input circuit signal to the "y" deflection circuitry. Normally, the vertical input circuitry signal ("y") is used for triggering the trace; but most scopes make an external input available to the trigger circuit, allowing an external event to cause triggering, thereby synchronizing the trace to the external event.

The z axis is the intensity circuit that can vary the brightness of the trace. The z axis is normally available with an external input that allows a signal to enhance (or blank) any portion

of the displayed signal occuring during the z axis signal.

Listed controls are those found on almost all laboratory oscilloscopes. There are many other control functions available, and they differ from manufacturer to manufacturer and from model to model. A thorough reading of the operation manual for the oscilloscope used is recommended for successful operation.

SINUSOIDAL WAVE ANALYSIS

The pictorial display presented on the oscilloscope by a wave pattern allows measurements of its amplitude and frequency. In addition, characteristics such as linearity, symmetry, and other parameters of the signal are available for diagnosis. Figure 5 represents a typical sinewave (sinusoidal) display on an oscilloscope. The settings of the oscilloscope controls are listed beside the display. These settings are used to determine amplitude and frequency of the wave as follows.

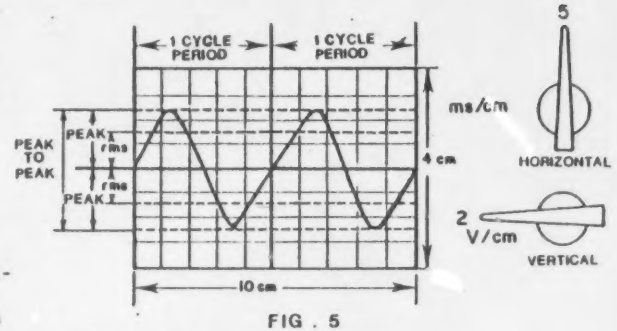


FIG . 5

The vertical attenuator setting is 2 V/cm and the horizontal setting is 5 ms/cm. Since peak-to-peak voltage can be observed to span 5 cm, the peak-to-peak voltage is 10 V. The peak voltage is 5 V (2.5 cm) and rms (effective) voltage can be obtained by multiplying peak voltage by 0.707, or approximately 3.5 volts.

Frequency of the input signal is determined by counting the number of centimeters spanned by a full cycle of input signal. In Figure 5, a full cycle (one point on the signal to the next corresponding point on signal) takes up 5 centimeters. Thus, the period of the wave is 5 cm x 5 ms/cm or 25 ms. Frequency is determined by taking the reciprocal of the period, which results in 1/0.025 = 40 Hz. Thus, the signal frequency is 40 Hz.

The real value of the oscilloscope becomes apparent when signals other than those of the simple sinusoidal form are displayed. Figure 6 shows some complex waves that can be analyzed from the scope display. Even for each complex wave forms, the oscilloscope presents a display that gives the following information about the signal(s): (1) whether or not the signal is periodic (repeats); (2) signal amplitude; and (3) signal repetition rate (frequency). If several signals are admitted together, the scope displays the resultant, or sum, of the individual signals immediately.

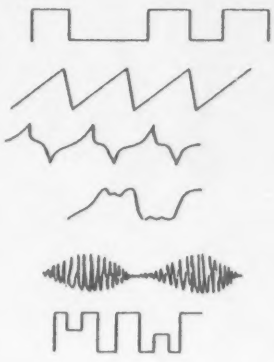


FIG . 6

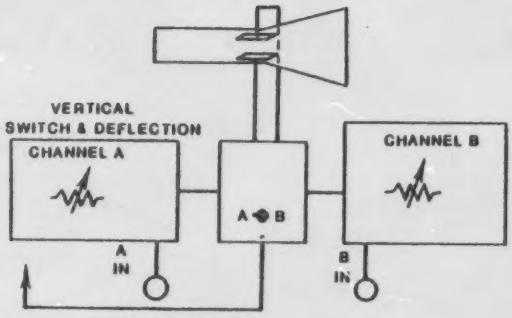


FIG . 7

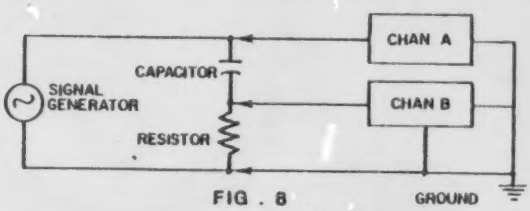


FIG . 8

Such resultants might otherwise require a great deal of mathematics to determine. All the wave forms shown in Figure 6 can be considered as a sum of sinusoidal waves of different amplitudes and frequencies.

PHASE COMPARISON

A duel-trace scope can be used to determine the phase relationship of one signal to another. A dual-trace scope has the same general block diagram as the basic oscilloscope, except that another vertical channel is provided; and a switching circuit is able to display alternately one channel and then the other, giving the illusion of two signals simultaneously displayed. A simplified block diagram is shown in Figure 7. In the circuit shown schematically in Figure 8, the phase difference between the applied voltage and the current in an RC (resistance-capacitance) circuit can be observed. The oscilloscope is connected in such a manner that voltage across the capacitor-resistor combination goes to channel A, and voltage across the resistor (in phase with the current) goes to channel B. Since voltages in an a.c. circuit generally are not in phase, the dual trace oscilloscope can be used to determine phase differences by permitting measurement of the relative shift of the sine waves.

If, for example, resistance, R, or the resistor (Figure 8) equals capacitive reactance ($\frac{1}{2\pi fc}$) of the capacitor, there will be a 45° phase shift between applied voltage and voltage across a pure resistance. Note: Caution must be observed, in that most oscilloscopes have a common, or chassis, ground (as do most signal generators). When connected in a signal circuit, care must be taken to ensure that all ground points correspond to the ground reference point of the circuit.

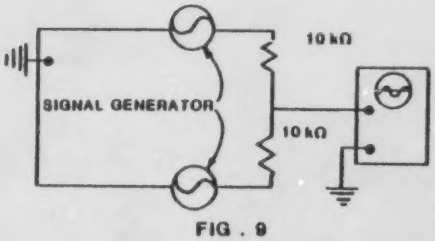


FIG . 9

When a single trace with two mixed sources is used, an algebraic result can be represented. Figure 9 shows the mixing of two frequency sources. Patterns of reinforcement, cancellation, and harmonic interference can be observed with this method. Reinforcement occurs when both signals are in phase. Cancellation occurs when both signals are out of phase.

LISSAJOUS PATTERNS

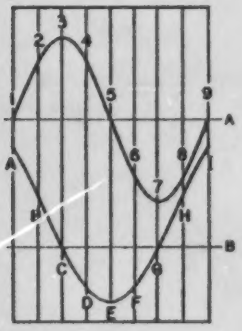


FIG. 10

Lissajous patterns are a method of determining frequency of a signal in comparison to a known one, and to some extent, the phase of a signal in comparison to an unknown signal of the same frequency.

Figure 10 shows two signals of the same amplitude and frequency, but 90° out of phase. When one signal (A) is applied to the y-deflection plate and the other (B) to the x-deflection plate (x-y position of time/cm switch or external horizontal input), the display shown in Figure 11 is obtained. Note that x and y deflections in the circular pattern can be related at each point to deflections shown in the A and B traces. One revolution of the Lissajous pattern corresponds to one period of the waves. If the same frequency signals are in phase (Figure 12a) they give a Lissajous pattern (Figure 12b). If identical frequency signals are 180° out of phase (Figure 13a), they result in a Lissajous pattern (Figure 13b).

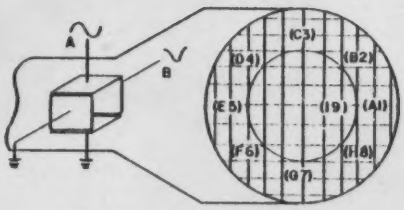


FIG. 11

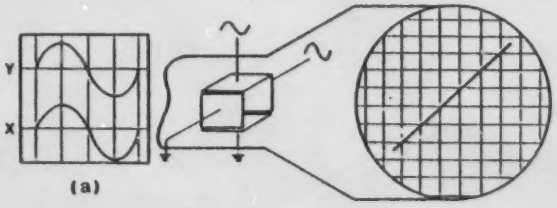


FIG. 12

More interesting patterns result if the two perpendicular signals are of different frequencies, especially when their ratio is a whole number. Figures 14a and 14b show two sets of such signals, the known signal going to the horizontal plates and the unknown signal going to the vertical plates.

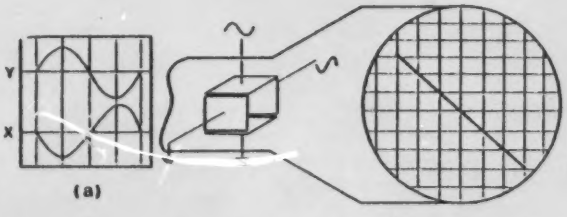


FIG. 12

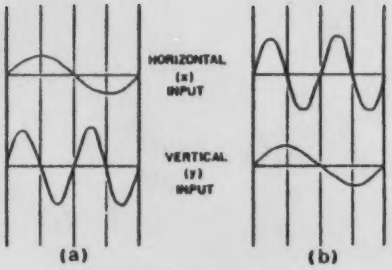


FIG . 14

Figures 15a and 15b show the resultant Lissajous patterns. In one case, the unknown signal is twice the horizontal (a); and in the other case, the unknown signal is one-half the horizontal frequency (b). Such patterns are particularly useful in determining frequencies out of the range of the signal generator.

There are many Lissajous patterns; Figures 16a and 16b show several of them, plus methodology for determining input frequency from the pattern. In Figure 16a, unknown is submultiple of known (x) input. The unknown is, therefore, a lower frequency than known input. Count number of loops vertically on one side and divide into standard frequency. Count number of loops horizontally (at top or bottom) and multiply by standard frequency. In Figure 16b, unknown is multiple (higher frequency) of known (x) input. Count number of loops horizontally and multiply by standard frequency. Count number of loops vertically on one side and divide into standard frequency.

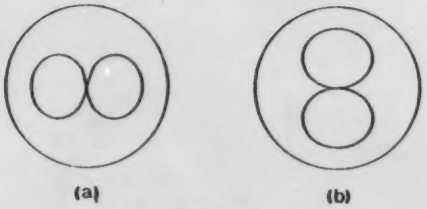


FIG . 15

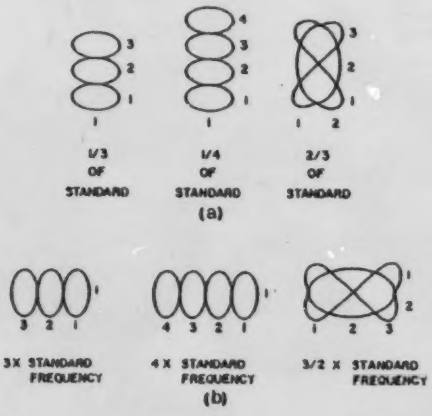


FIG . 16

SUMMARY

The oscilloscope is a device that utilizes a cathode-ray tube to produce a visual display of the waveforms of electromagnetic phenomena. It is one of the most vital and convenient instruments for the measurement of electrical quantities. The extreme frequency range is one of its principal features; that is, oscilloscopes are capable of displaying voltages having frequencies of hundreds of megahertz. Photographs often are taken of the data displayed.

SINEWAVE ANALYSIS

MATERIALS

Ballatine 1010A dual-trace oscilloscope (or equivalent scope): d.c.-250 kHz minimum vertical response, triggered time base with calibrated horizontal sweep having a minimum of 1 μs/cm (microsecond/centimeter) trace time. The screen requires a 8 x 10 graticule.

Signal generator. A 30 Hz to 300 kHz minimum, controllable (attenuated output) with at least ±1 volt rms output. (Two of these generators will be required for Procedure 3.)

PROCEDURE

Set up circuit (Figure 17). Set signal generator to 1000 Hz; set oscilloscope attenuator to 0.1 V/cm; set horizontal time base to 0.1 ms/cm. Adjust trigger control to obtain a steady display. Adjust signal generator output until display amplitude just fills four centimeters (or four divisions).

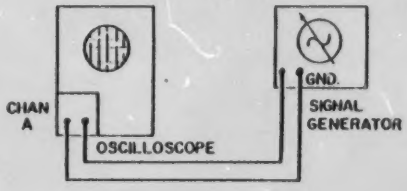


FIG. 17

1. Peak voltage is _____ volts.

2. Peak-to-peak voltage is _____ volts.

3. The rms voltage is _____ volts.

Adjust signal generator frequency control so signal period just fills 7 centimeters.

Compute frequency (1/period): _____ hz.
What frequency does generator indicate? _____ Hz.

Normally, calibrate time base of oscilloscope is more accurate than signal generator. (However, the laboratory standard signal generator might be more accurate.)

PHASE DIFFERENCE

MATERIALS

Same as Procedure 1, and further including:

0.01 μF capacitor (molded mylar), 100 V d.c.
2 kΩ (2000 ohm) variable potentiometer.

PROCEDURE

Assemble circuit (Figure 18).

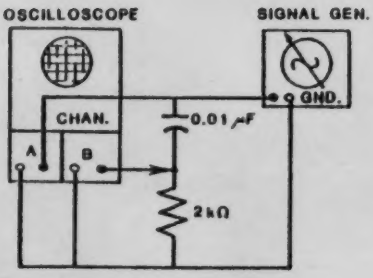


FIG. 18

1. Set variable resistor to mid-range.

2. Set signal generator for 10 kHz; use dual trace, and display on channel A the voltage across capacitor-resistor combination. Triggering should be taken off channel A. Channel B should display voltage across resistor.

3. Set vertical deflection on channel A for 0.5 V/cm, and adjust signal generator output to display four centimeters of amplitude.

4. Adjust vertical deflection on Channel B for a viewable signal, preferably four centimeters in amplitude; and display channel B signal below channel A signal on screen.

5. Slightly vary potentiometer. Notice that the phase between the signal on A and signal on B will change. Extreme rotation of control to low resistance will decrease viewable signal beyond usefulness. Use only midrange (1000 ohms) to maximum resistance (2000 ohms). When $X_C = R$ (where X_C = capacitive reactance and R = resitance), there should be a 45° phase difference between both signals.

LISSAJOUS PATTERNS

MATERIALS

Same as Procedure I, except that two signal generators are required.

PROCEDURE

Assemble equipment (Figure 19).

1. Set both signal generators to approximately the same output signal amplitute and frequency.
2. Set time/cm switch to x-y position (or use external horizontal input on some scopes).
3. Adjust "y" input attenuator until a roughly square display is obtained.
4. Slightly vary frequency control of one signal generator until a circular trace occurs. This trace means that the two generators are producing the same frequency, but are 90° out of phase. In all probability, the circle will vary from a 45° line running up from left to right (180° out of phase) through 45° running up from right to left (in phase).
5. Adjust signal generator to a multiple of "x" input (horizontal) generator. When multiple is reached, notice pattern reproduced.
6. Try to obtain all patterns shown in Figure 16.

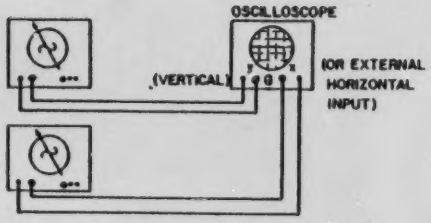


FIG . 19

SELF CHECK

1. Vertical control operates on (x)(y) axis?
2. Horizontal control operates on the (x)(y) axis?
3. The z axis control affects the _____ of the display.

4. Time per centimeter refers to which of the below items?

   Entire horizontal axis.
   Entire vertical axis.
   One vertical division.
   One horizontal division.

5. A _____ type oscilloscope is required to measure phase difference.

6. The trigger control affects the _____ circuit.

7. With time/cm control set to 0.2 milliseconds per cm, a waveform takes 7.5 cm to complete one cycle. What is its frequency?

8. With V/cm control set to 0.2 V/cm and a display that fills 6.7 cm, what is the voltage displayed?

9. What precaution concerning grounding of test equipment is necessary in making a proper measurement?

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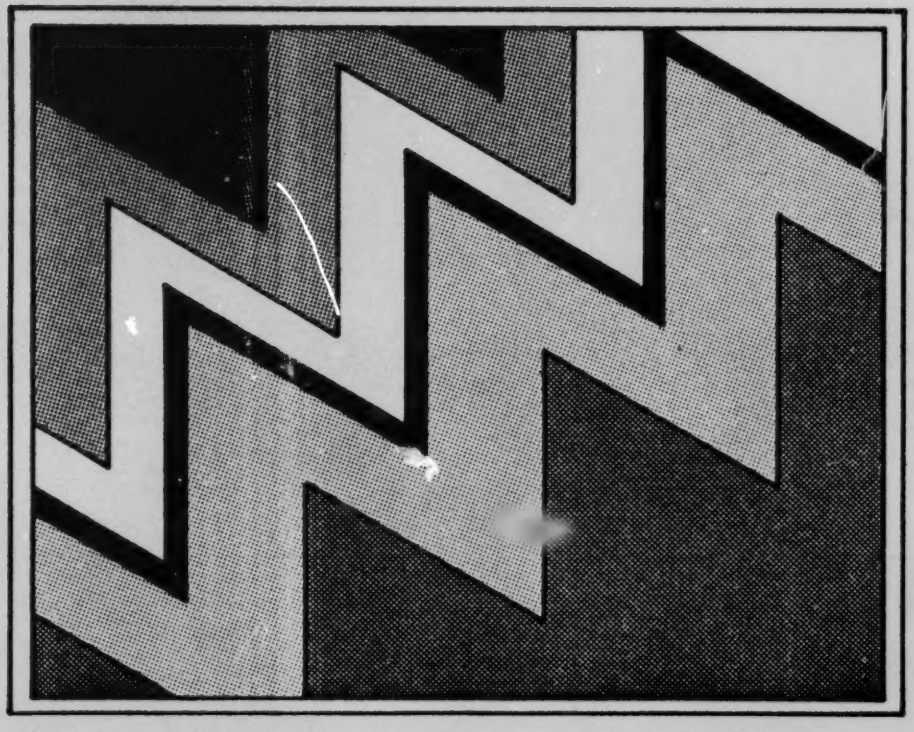
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 11 E 4

RECTIFIERS



ELECTRICAL APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center – Southwest

TERC – SW

INTRODUCTION

The conversion of alternating current into direct current is necessary in many electronic circuits. A rectifier (diode) accomplishes this conversion. In this module, the principle of rectification will be discussed, and operation of a rectifier in an electrical circuit will be examined. Rectification in fluid and mechanical systems also will be discussed. In the laboratory, the student will assemble circuits containing rectifiers, and will examine their performance with an oscilloscope.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define, in two or three sentences, an electronic rectifier.
- Draw and label graphs as functions of time for the following:

  Sine wave signal applied to a rectifier and resistor in series.
  Voltage across rectifier.
  Voltage across resistor.
  Current through circuit.

- Describe a bridge rectifier circuit (include a diagram with identification labeling of major items such as a.c. signal source and load resistor).
- Describe a full-wave rectifier circuit for use with a center-tapped transformer (include a diagram with identification labeling or major items such as transformer and load resistor).

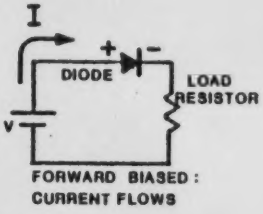
- Give an example of rectification in:

  A fluid system.
  A mechanical system.

- Given the appropriate equipment, construct half-wave and full-wave rectifiers, and measure time-varying voltages on each component with an oscilloscope.

ELECTRONIC RECTIFIERS

In electronic terms, a rectifier (also called a diode) is a device that allows a current to flow in one direction only. Figure 1 shows two circuits, one containing a forward- and the other a reverse-biased rectifier. The electronic symbol for a rectifier is an arrow with a vertical bar at the point (→|). The bias of a rectifier is the direction of potential difference across the rectifier. When the back of the arrow is made positive and the point is made negative, the rectifier is forward biased (+ →| -) and represents a low resistance to current flow in the forward direction. A small voltage (approximately 2.3 volts for a solid state silicon rectifier) appears across the diode. The remainder of the applied voltage appears across the resistor.



When the diode is reverse biased, the back of the arrow is negative and the point is positive (- →| +). In this case, the diode presents an almost infinite resistance and effectively blocks current flow in the circuit. In this direction, most of the voltage appears across the diode. As the resistance is not actually infinite, a small current does flow in the circuit. This is called the leakage current. It is so small, however, that it usually can be ignored.

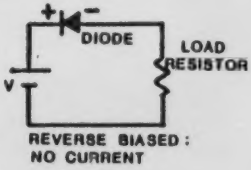


FIG. 1

In the remainder of this module, the forward resistance of diodes will be treated as zero; and the reverse resistance, infinite. This treatment will give a voltage of zero on the diode when it is forward biased and a voltage equal to that of the source when the diode is reverse biased. This treatment of an ideal diode introduces little error in most circuits.

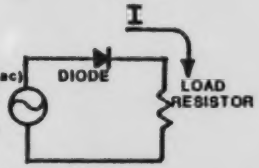


FIG. 2

Figure 2 shows a typical circuit containing a.c. input signal diode and load. Figure 3 shows the time-varying voltages and currents in diode circuit shown in Figure 2. The source voltage is a sine wave that results in an alternating forward and reverse bias on the diode. When the upper terminal of the a.c. source is positive, the diode is forward biased and has zero resistance. The total voltage will appear across the resistor and will be, at any instant, the product of current in the circuit and resistance of the load resistor. The current direction will be that shown in Figure 2.

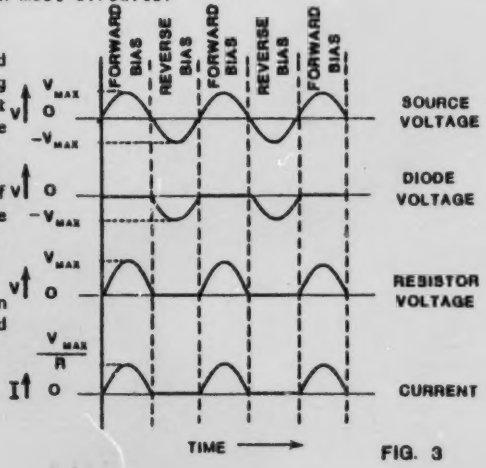


FIG. 3

When polarity of the voltage source (V_{ac}) reverses, the diode be-

comes reverse biased and provides an infinite resistance. The voltage of the source now appears across the diode. The resistor voltage is zero and the current in the circuit is zero. The variation of current with voltage is shown in Figure 3.

The circuit discussed above is called a half-wave rectifier circuit because the applied signal voltage appears on the load resistor only half the time - during the forward-bias portions of the cycle. This voltage is always positive, although not constant in magnitude, and, therefore, is "rectified" in the form of a pulsating d.c. voltage.

In many applications, the transmission of all applied voltage to the load, and the rectification of it at the same time, is desirable. This procedure is accomplished with the full-wave bridge rectifier circuit incorporating four diodes (Figure 4.)

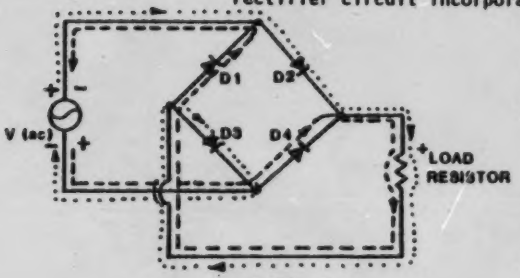


FIG. 4

During the half cycle when the upper terminal of the source is positive, D2 and D3 are forward biased and D1 and D4 are reverse biased. Current flows from the upper terminal of the source through D2, through the load resistor, through D3 and to the lower terminal of the source, as indicated by the dotted line in Figure 4. During this half cycle, D1 and D4 block current flow. During the following half cycle when the lower terminal of the source becomes positive, the bias direction changes for each diode. Current then flows from the lower (positive) terminal of the source through D4, through the resistor, through D1 and to the upper terminal of the source (dashed line). Direction of current flow through the resistor is the same in both half cycles. Figure 5 shows the resulting voltage on the load resistor in Figure 4. The rectified voltage is again a pulsating d.c. voltage, this time without missing half cycles.

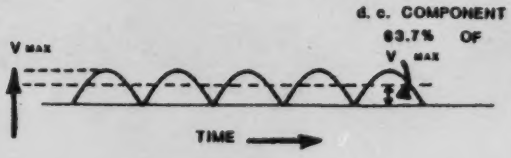


FIG. 5

Figure 6 shows another kind of full-wave rectifier circuit, used with a center-tapped transformer and only two diodes. When point A is positive, D1 is forward biased and current flows as indicated by the dotted line. Because point C is negative, D2 is reverse biased and no current flows through it. On the next half cycle, point A becomes negative and point C becomes positive. D2 is then forward

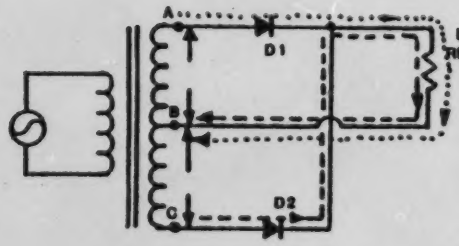


FIG. 6

biased, allowing current to flow as shown by the dashed line. The result is again the full-wave rectified voltage across the resistor shown in Figure 5. This is not a pure d.c. voltage. It can be thought of as containing both d.c. and a.c. components, although the result is never negative. For full-wave rectification, the d.c. component is 63.7% of the maximum voltage. For half-wave rectification, the d.c. component is half this, or 31.8% of V_{max}.

Rectifiers are used in electronic circuits when a.c. currents and voltages must be converted to d.c. This need arises most commonly in power supplies for electronic equipment, in logic and switching circuits, and in demodulation circuits for communication. The principle of rectification is also applied in other energy systems.

MECHANICAL RECTIFIERS

Figure 7 shows a check valve (rectifier) for use in a fluid system. When fluid pressure is applied from the spring end of the valve, both spring and pressure force the ball into the seat; and no flow is present. When pressure is applied from the ball end of the valve, the spring is compressed and fluid passes around the ball. The valve then behaves like a fluid-system diode, allowing flow in one direction only.

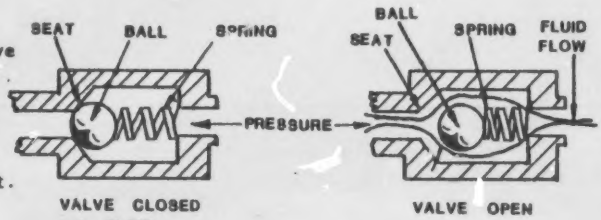


FIG. 7

Figure 8 shows operation of a ratchet (rectifier) in a mechanical system. The hinge in the lever is designed to bend upward only, and the ratchet has a stop that will allow it to swing upward only. When a downward force is applied to the lever, the rack moves upward. The ratchet swings upward to allow one tooth of the rack to pass. When the free end of the lever is moved upward, the ratchet prevents a downward motion of the rack. The hinged end of the lever moves downward past the next tooth of the rack, and the entire process then may be repeated. This rack and ratchet system then operates as a mechanical system diode, allowing mechanical motion in one direction only.

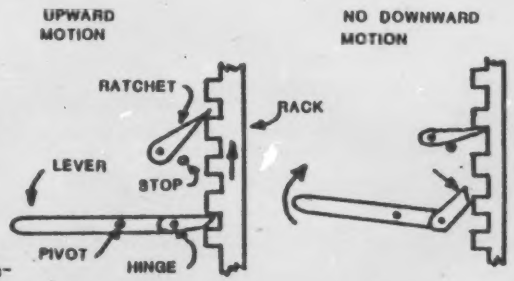


FIG. 8

SUMMARY

Rectifiers allow motion in only one direction. An electronic rectifier (diode) allows current to flow in one direction only, thereby converting alternating current into direct current. Mechanical rectifiers include the check valve (which permits fluid to flow in but one direction) and the ratchet (which limits mechanical motion to one direction).

PREFORMANCE OF RECTIFIER CIRCUITS

MATERIALS

a.c. power cord with clips attached.
Four diodes, 1 A, 100 PIV minimum.
Resistor, 2 W, 100C Ohms minimum.
Transformer, 24 V, 0.25 A minimum.
Clip leads.
Dual-trace oscilloscope.

Note: A single-trace oscilloscope may be used, but it will not show the phase relationship of rectifier and resistor voltages. These two signals must be viewed separately if a single-trace scope is used.

PROCEDURE

CAUTION: Dangerous voltages are present when this circuit is energized. Do not touch any components while power cord is plugged in. Disconnect power at end of each step.

1. Connect circuit as shown in Figure 9 (do not plug in power cord); set oscilloscope sensitivity for 10 V/cm; set time base for 5 ms/cm. Turn on oscilloscope and obtain a stable trace.

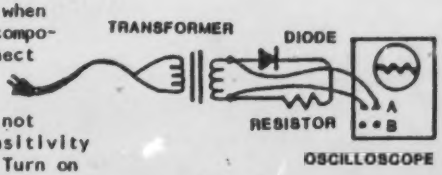


FIG. 9

2. Plug in power cord; observe signal on oscilloscope; sketch signal in Data Table. Unplug power cord.

3. Connect circuit as shown in Figure 10.

4. Plug in power cord; observe traces on scope. Trace A is diode voltage; trace B is resistor voltage. Sketch both signals in Data Table. Unplug power cord.

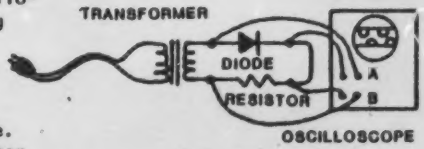


FIG. 10

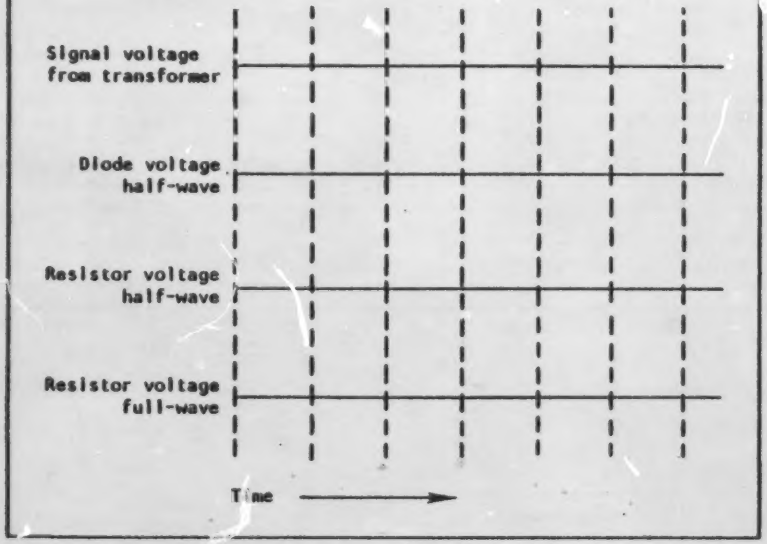
5. Connect circuit shown in Figure 11.

6. Plug in power cord; observe voltage on resistor and sketch in Data Table. Unplug power cord.

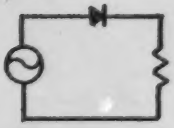
FIG. 11

7. What do the oscilloscope traces for each circuit indicate?

DATA TABLE



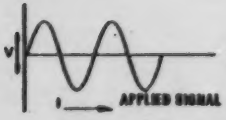
SELF CHECK



1. Define an electronic rectifier.

2. For the circuit at left, draw and label graphs as functions of time for:

   Voltage across rectifier.
   Voltage across resistor.
   Current through circuit.



3. Draw and label a diagram of a bridge rectifier circuit. Show current flow through the circuit for both the positive and negative half-cycle of the signal source.

4. Draw and label a full-wave rectifier circuit for use with

a center-tapped transformer. Show current flow through the circuit for both the positive and negative half-cycle of the signal source.

5. Draw and label diagrams illustrating an example of rectification in:

   A fluid system.
   A mechanical system.

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Malmstadt, H.V.; Enke, G.G; and Toren, E.C., Jr. Electronics for Scientists. New York: W.A. Benjamin, Inc., 1963.

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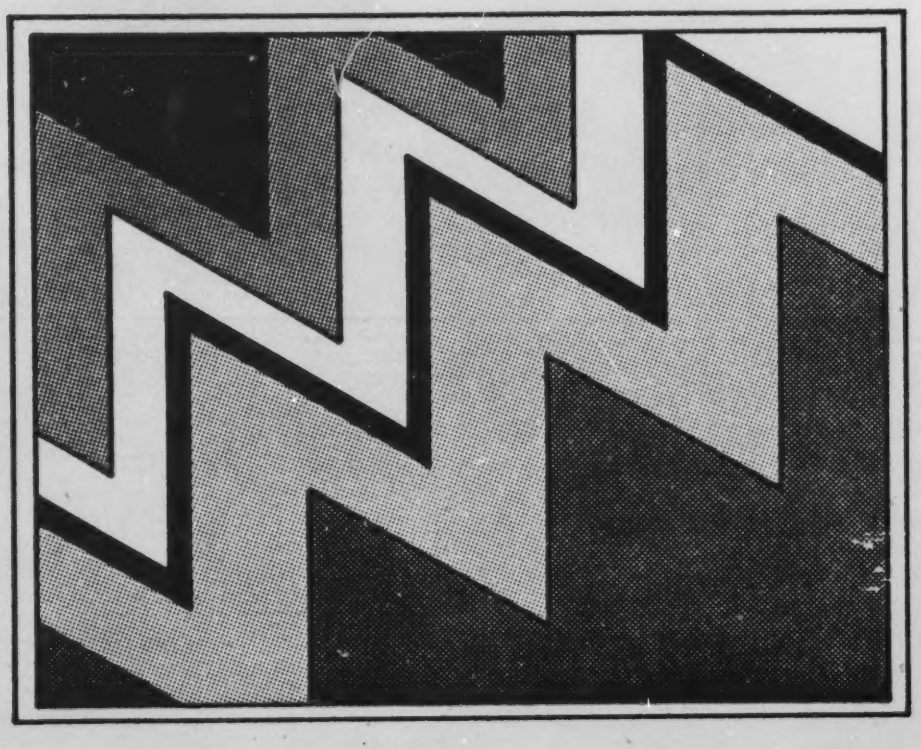
UNIFIED TECHNICAL CONCEPTS

MODULE 11 E 5

ELECTRONIC FILTERING IN D.C. POWER SUPPLIES



ELECTRICAL APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

Most electronic circuits operate on d.c. power, but most electrical power-distribution systems deliver a.c. power. Module IIE4, "Rectifiers," describes the operation of rectifier circuits used to convert a.c. electrical signals to pulsating d.c. signals. Pulsating d.c. voltage and current must be smoothed to a more constant d.c. level for most applications. This is accomplished by a suitable electronic filter. Several types of filters are used in d.c. power supplies; the type used depends upon the output characteristics required. This module discusses the construction, characteristics, and applications of several of the more common filter types. The student will construct and evaluate several filters in the laboratory.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves," and Application Module IIE4, "Rectifiers."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following terms:

  Ripple voltage.
  Ripple factor.
  Percent ripple.

- Describe the operation of a capacitor filter having a resistive load, making use of appropriate diagrams.

- Name at least four types of filters, and qualitatively compare the effectiveness of each filter with regard to ripple factor.

- Given the necessary data for a d.c. power supply, calculate the ripple factor and the d.c. output voltage for a C type filter and an RC π-section filter for a half-wave and a full-wave input voltage.

- Draw a complete electronic power supply consisting of a transformer, full-wave rectifier, RC (PI) type filter, and load; also describe and illustrate, by means of graphs, the output of the transformer, rectifier, and filter.

- Given the appropriate equipment, construct a half-wave and full-wave d.c. power supply with an RC type filter for different values of capacitors; then measure the output current, voltage, and ripple voltage of each, and make comparisons with the calculated values.

DISCUSSION

FILTER CONSIDERATIONS

Power-supply outputs, which usually are filter outputs, consist of a d.c. component and a relatively small a.c. component. In power-supply circuits, the a.c. component is called "ripple voltage" or simply "ripple." The output waveform of a d.c. power supply is depicted in Figure 1.

The ripple voltage with respect to the d.c. voltage usually is a significant property in comparing the effectiveness of a filter circuit. Another important property in the comparison of filter circuits is the degree of filter output-voltage change with variation in load current. This property, not discussed in this module, is called "regulation."

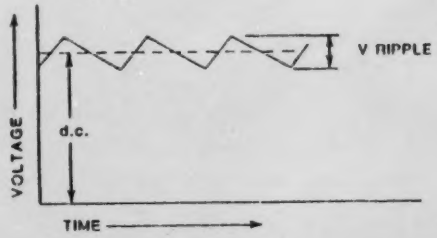


FIG. 1

There are many variations in filter configurations. Filter configurations are dictated by different requirements for output voltage and/or current, ripple voltage with respect to d.c. voltage, variations in output voltage with respect to variations in load current, cost, size, surge currents, etc.

RIPPLE FACTOR AND % RIPPLE

By definition, ripple factor is the ratio of the RMS value of a.c. components to the d.c. voltage; that is —

$$\text{Ripple Factor} = \frac{\text{RMS Value of Ripple Voltage}}{\text{d.c. Value of Voltage}}$$

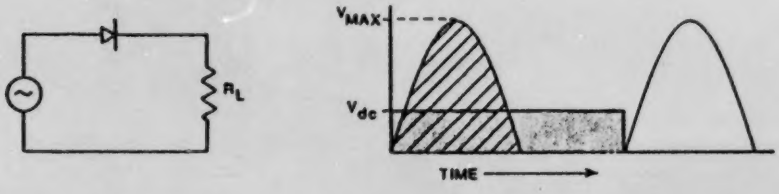
In comparing the effectiveness of a filter with respect to "smoothing" a.c. variations, the smaller the ripple factor, the better the filter circuits. The "percent ripple" is the ripple factor expressed as a percent:

$$\% \text{ Ripple} = \text{Ripple factor} \times 100\%$$

Again, the smaller the % ripple, the better the filter current with respect to filter-smoothing action.

THE CAPACITOR FILTER

An alternating signal such as that from 120-V, 60-cycle electrical outlet has zero average value and, therefore, has no d.c. value. If such a signal is rectified by means of a circuit, as in Figure 2, the output signal does have a d.c. value, although it is not constant.



(a) (b)

FIG. 2

The average voltage V_{dc} is represented by the height of a rectangle having the same area as the area under the signal in the interval of one cycle of the applied a.c. wave. The d.c. voltage, the RMS value of the a.c. component of the rectified wave, and the ripple factor r have the following values:

$$V_{dc} = \frac{V_{max}}{\pi} = 0.318\ V_{max}$$

$$V_{rms} = 0.385\ V_{max}$$

$$r = \frac{V_{rms}}{V_{dc}} = \frac{0.385\ V_{max}}{0.318\ V_{max}} = 1.21$$

This relationship indicates that the ripple exceeds the d.c. voltage level.

This value for ripple factor can be reduced greatly by the addition of a large capacitor in parallel with the load resistor, as illustrated in Figure 3a. The value of this capacitor must be large enough to result in an RC time constant that is large compared to the period of the input signal. The output voltage of the capacitor filter is shown in Figure 3b. The capacitor quickly charges to the value of the input signal; it then discharges slowly through the load resistor. The "filter output" curve of the figure represents both output voltage and current through the load resistor. A small-value resistor R_s is placed in series with the input voltage to reduce the surge current, which otherwise may damage the diode.

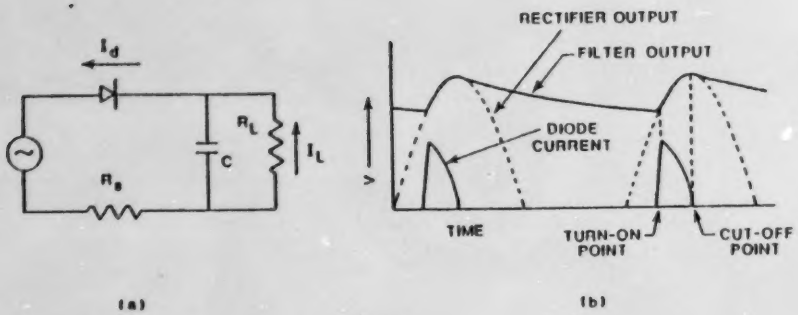


FIG. 3

The action of this circuit can be described as follows: (1) When the output voltage of the rectifier is increasing, the capacitor will charge to a voltage equal to the output of the rectifier. (2) As the rectifier potential falls below that of the capacitor, the capacitor delivers current to the load, thus maintaining the output voltage at a high level for a longer period of time than if the capacitor were not in the circuit. (3) As the output of the rectifier rises once more to a voltage equal to that remaining on the capacitor, the diode will start conducting. The current conduction becomes instantaneously equal to the load current. To understand this, note that when the capacitor voltage is equal to the rectifier-voltage output, the capacitor neither is charging nor discharging; however, because the resistor has the same value of current, all the load current must be supplied by the diode. The output d.c. voltage of the capacitor filter in Figure 3a is given by Equation 1.

$$V_{dc} = 1.4\ V_{rms} - \frac{I_L}{2fC} \qquad (1)$$

where

V_{dc} = average d.c. output voltage.
V_{rms} = RMS voltage of the input signal.
I_L = current through load resistor.
C = capacitance of filter capacitor.
f = frequency of applied a.c. signal.

The ripple factor for a 60-Hz input a.c. signal is given by Equation 2.

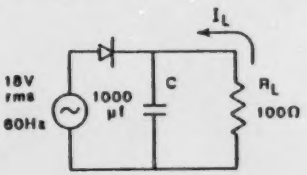
$$r = \frac{1}{210\ CR_L} \qquad (2)$$

Equations 1 and 2 are based upon approximations and should not be applied in cases in which the ripple factor exceeds 0.10. Example A illustrates the use of these equations in solving a problem.

EXAMPLE A: OUTPUT VOLTAGE AND RIPPLE FACTOR OF A CAPACITOR FILTER

Given: A power supply has an input of 18 V_{rms}, 60 Hz. The values o resistance and capacitance are —

$C = 1000\ \mu f$, $R_L = 100\ \Omega$.



Find: (1) Output voltage,
(2) ripple factor, and
(3) ripple voltage.

Solution: Estimate I_L by dividing V_{max} by R_L.

$$V_{max} = 1.4\ V_{rms}$$

$$= (1.4)(18\ V)$$

$$V_{max} = 25.2\ V$$

$$I_L = \frac{V_{max}}{R_L}$$

$$= \frac{25.2\ V}{100\ \Omega}$$

$$I_L = 0.252\ A$$

(1) Determine output voltage from Equation 1.

$$V_{dc} = 1.4\ V_{rms} - \frac{I_L}{2fC}$$

$$= (1.4)(18\ V) - \frac{0.252\ A}{(2)(60/s)(10^{-3}f)}$$

$$= 25.2\ V - 2.1\ V$$

$$V_{dc} = 23.1\ V$$

(2) Determine ripple factor from Equation 2.

$$r = \frac{1}{210\ CR_L}$$

$$= \frac{1}{(210)(10^{-3})(100)}$$

$$r = 0.046$$

(3) Ripple voltage (RMS).

$$V_r = rV_{dc}$$

$$= (0.0476)(23.1\ V)$$

$$V_r = 1.1\ V$$

The discussion thus far has been based upon the use of a single diode to produce half-wave rectification as illustrated in Figure 3. Figure 4a displays a power supply containing a full-wave bridge rectifier circuit that produces the output waveform shown in Figure 4b. The output frequency of a full-wave rectifier is twice the input frequency. For a full-wave rectifier, the frequency used in Equation 1 should be twice the a.c. input frequency to the rectifier. The ripple decreases proportionately, and a factor of 420 (instead of 210) is used in Equation 2. Thus, full-wave rectification will produce a ripple voltage half that present with half-wave rectification.

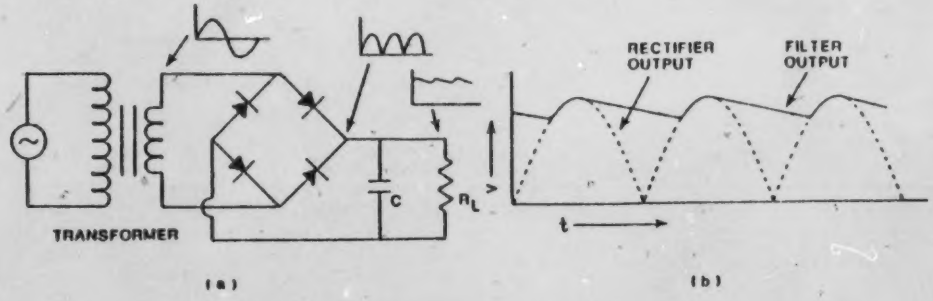
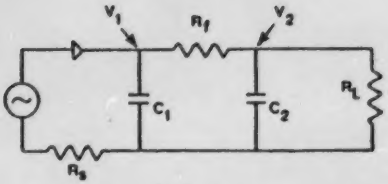


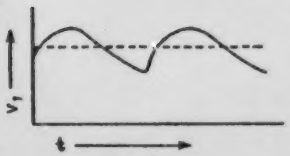
FIG. 4

RC Pi - SECTION FILTER

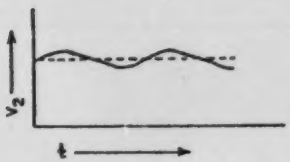
A decrease of the ripple voltage of the capacitance filter can be achieved by the addition of an RC network (Figure 5). In this circuit, C_2 is charged by the discharge current of C_1 through the filter resistor R_f. This process reduces the ripple voltage, as indicated in Figure 5a and 5b. Such reduction has the disadvantage of decreasing the regulation of the filter output. Because of the increased resistance of the filter, the output voltage will vary more with changes in output current than in the case of the



(a)



(b)



(c)

FIG. 5

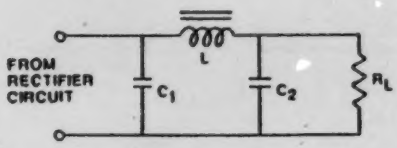


FIG. 6

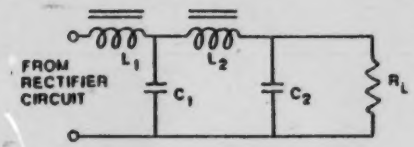


FIG. 7

capacitance filter. For this reason, RC π-section filters frequently are used in constant-load circuits or in circuits that do not require good regulation. RC filters have the additional disadvantage of dissipating large amounts of power in the filter resistor if the current is large. This shortcoming limits their use in high current applications.

OTHER TYPES OF FILTERS

From the above discussion of the C type and RC filters, it is apparent that a maximum reduction in ripple voltage with a minimum change in d.c. output voltage for changes in load current is the desired goal of all filter circuits. This should be accomplished with a minimum d.c. filter resistance to reduce power losses.

The use of properly-selected inductors in place of filter resistors provides a high series a.c. impedance, while providing a low d.c. series resistance (the internal resistance of the inductor). The combined effect of a low d.c. resistance and a high a.c. impedance results in better filter action with respect to both ripple factor and regulation. A typical LC filter circuit is shown schematically in Figure 6.

As evident in all filter circuits discussed so far, the first component "seen" by the rectifier output is a shunt capacitor. As mentioned earlier, to avoid the high-surge current that may damage the diode, a small series resistor R_s is placed in the input side of the filter. If a properly-selected inductor L_1 is used instead (Figure 7), the inductor will limit the current and, at the same time, will provide additional reduction in ripple voltage by offering high series a.c. impedance and low d.c. series resistance.

FILTER EQUATIONS

The mathematical development of filter equations is beyond the scope of this module. The equations for output voltage and for ripple factor for most common filter

types using both half-wave and full-wave rectification are given in Table 1. These equations are based upon a 60-Hz input to the rectifier and are useful for ripple factors under 0.10.

TABLE 1: POWER-SUPPLY EQUATIONS FOR 60-Hz RECTIFIER INPUT

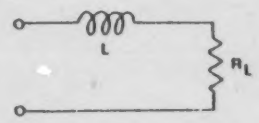
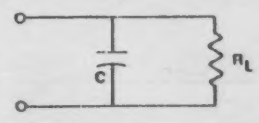
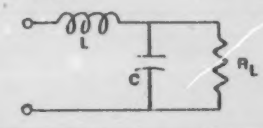
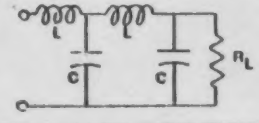
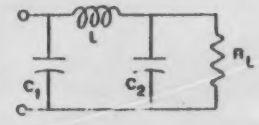
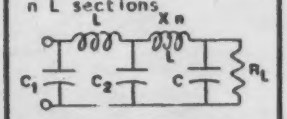
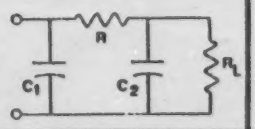
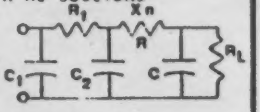
	Half-wave rectifier	
Filter Type	Output voltage	Ripple factor
L 	Do not use	Do not use
C 	$1.4\ V_{rms} - I_L/120\ C$	$1/210\ CR_L$
L section 	Do not use	Do not use
n L sections 	Do not use	Do not use
π section 	$1.4\ V_{rms} - \frac{I_L}{120\ C_1}$	$2.6 \times 10^{-8}/C_1C_2L;R_L$

Table 1. Continued.

π section plus n L sections 	$1.4\ V_{rms} - \frac{I_L}{120\ C_1}$	$(\frac{2.6 \times 10^{-8}}{C_1C_2L_1R_L})(\frac{7 \times 10^{-6}}{LC})^n$
π-section RC 	$1.4\ V_{rms} - I_L\ (\frac{1}{120\ C_1} + R)$	$10^{-5}/C_1C_2RR_L$
π-section RC plus n RC sections 	$(1.4\ V_{rms} - I_L)(\frac{1}{120\ C_1} + R_1 + nR)$	$(\frac{10^{-3}}{C_1C_2RR_L})(\frac{2.6 \times 10^{-3}}{RC})^n$

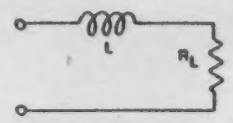
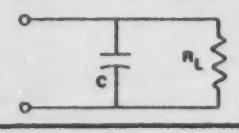
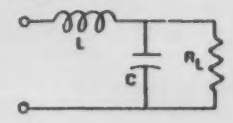
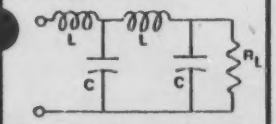
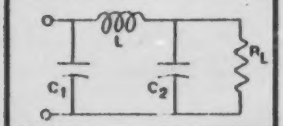
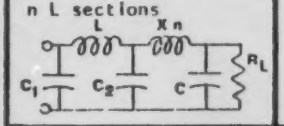
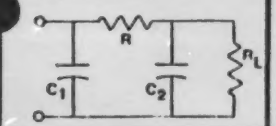
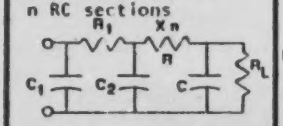
	Full-wave rectifier	
Filter type	Output voltage	Ripple factor
L 	$0.90\ V_{rms}$	$R_L/1600L$
C 	$1.4\ V_{rms} - I_L/240C$	$1/420\ CR_L$
L section 	$0.90\ V_{rms}$	$8.3 \times 10^{-7}/CL$

Table 1. Continued.

n L sections 	0.90 V_{rms}	$(0.47)(\frac{1.76 \times 10^{-6}}{LC})^n$
π section 	1.4 V_{rms} - $I_L/240\ C_1$	$3.3 \times 10^{-9}/C_1C_2L_1R_L$
π section plus n L sections 	1.4 V_{rms} - $I_L/240\ C_1$	$(\frac{3.3 \times 10^{-9}}{C_1C_2L_1R_L})(\frac{1.76 \times 10^{-6}}{LC})^n$
π-section RC 	1.4 V_{rms} - I_L $(\frac{1}{240\ C_1} + R)$	$2.5 \times 10^{-6}/C_1C_2RR_L$
π-section RC plus n RC sections 	$(1.4\ V_{rms} - I_L)(\frac{1}{240\ C_1} + R_1 + nR)$	$(\frac{2.5 \times 10^{-6}}{C_1C_2R_1R_L})(\frac{1.3 \times 10^{-3}}{RC})^n$

SUMMARY

Electronic filters are used in d.c. power supplies to reduce the ripple of a.c. components at the load. Capacitors store energy during the voltage peaks in the rectifier output. This stored energy is released slowly to maintain load current during the low-voltage portion of the rectifier output. A wide variety of filter types is employed, depending upon the ripple factor, regulation, and current required of the power supply. Filters that contain inductors provide lower ripple, better regulation, and lower power losses, but do so only at increased cost.

ELECTRONIC FILTERING IN D.C. POWER SUPPLIES

MATERIALS

Transformer with power cord, 24 V, 0.25 A minimum.
Four rectifiers, 1A, 100 PIV minimum.
Two pairs of filter capacitors, 35 V minimum, 200 μf one pair, approximately double the other pair.
Load resistor, R_L, 2 W, 560 OHMs.
Resistor, R_f, 1 W, 56 OHMs.
Resistor, R_s, 1/2 W, 5.6 OHMs.
Multimeter.
Oscilloscope.

PROCEDURE

SAFETY PRECAUTION: Voltages that are dangerously high are present when the circuit is energized. Disconnect power before touching any component in the circuit. Capacitors continue to be energized some time after power is disconnected. Handle them with care!

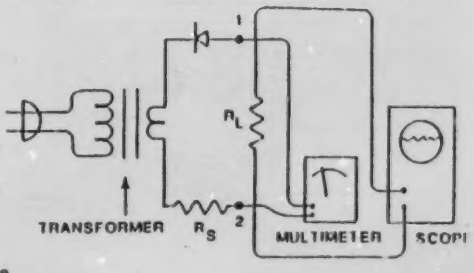


FIG. 8

1. Construct the circuit shown in Figure 8. Have instructor check circuit before connecting power to it.

2. Measure d.c. voltage V_{DC} with multimeter, and measure peak-to-peak ripple voltage with oscilloscope. Record these measurements in Data Table 1. Observe wave form on oscilloscope and make a drawing of it.

3. Replace R_L with circuit illustrated in Figure 9, and repeat step two. The circuit in Figure 9 is connected across points 1 and 2 in Figure 8.

4. Use a higher value for C_1, and repeat step two.

5. Repeat Step 2 for circuit in Figure 10, using the set of capacitors with lower value.

6. Use the higher-value capacitor C_1 and C_2 in Step 5, and repeat Step 2.

7. Repeat Steps 1 through 6, using full-wave rectifier depicted in Figure 11. Record results in Data Table 2.

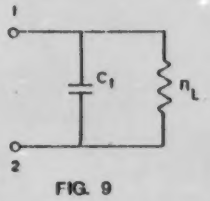


FIG. 9

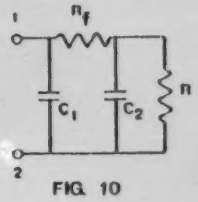


FIG. 10

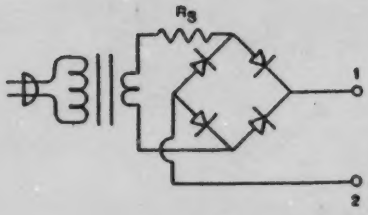


FIG. 11

8. Use the equations given in Table 1 to calculate the d.c. output voltage and peak-to-peak ripple voltage expected for each filter; enter results in Data Tables 1 and 2. The peak-to-peak ripple voltage is found by multiplying the RMS ripple voltage by 2.8.

9. Write a paragraph comparing the ripple voltages of all filters tested.

DATA TABLE 1					
HALF-WAVE					
		Calculated		Measured	
Step	Filter	V_{dc}	V_r (P-P*)	V_{dc}	V_r (P-P*)
1 & 2	None				
3	1				
4	2				
5	3				
6	4				
* (P-P) ≡ peak-to-peak					

Sketch the waveform observed with no filter:

Draw each filter used, and label with values of all components.
Sketch the waveform observed for each filter.

Filter 1

Filter 2

Figure 3

Filter 4

DATA TABLE 2					
FULL-WAVE					
		Calculated		Measured	
Step	Filter	V_{dc}	V_r (P-P)	V_{dc}	V_r (P-P)
1 & 2	None				
3	1				
4	2				
5	3				
6	4				

Sketch the waveform observed with no filter:

Draw each filter used, and label with values of all components. Sketch the waveform observed for each filter.

Filter 1

Filter 2

Filter 3

Filter 4

SELF CHECK

1. Define the following terms:

   a. Ripple voltage.
   b. Ripple factor.
   c. Percent ripple.

2. Describe the operation of a capacitor filter having a resistive load and half-wave rectification. Include diagrams that illustrate rectifier output voltage, filter output voltage, and diode current.

3. Sketch the following filters:

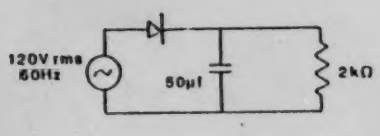
   a. C-filter.
   b. π-section RC.
   c. L section.
   d. π-section LC.

4. Describe the advantages of LC filters over RC filters.

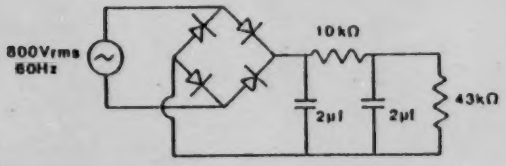
5. Draw a complete electronic power supply circuit consisting of a transformer, a full-wave rectifier, and a π-section RC filter. Show the waveform at the output of the transformer, after rectification, and at the load resistor.

6. Calculate the output d.c. voltage and the ripple factor of the following filter circuits.

a.



b.



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Faber, Rodney B. Applied Electricity and Electronics for Technology. New York: John Wiley & Sons, 1978.

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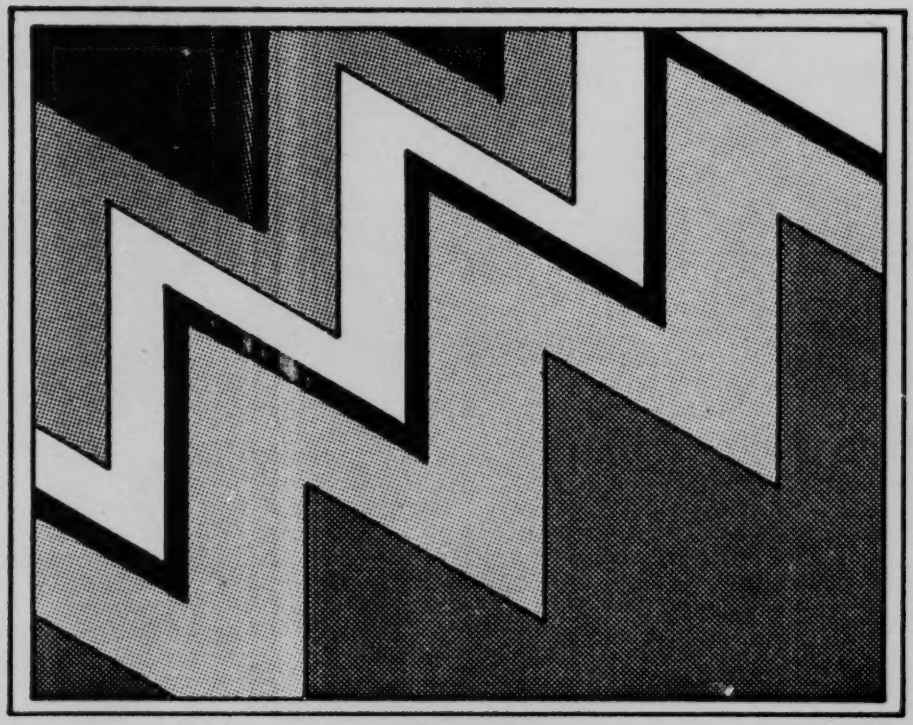
UNIFIED TECHNICAL CONCEPTS

MODULE 11 E 6

ELECTRONIC FILTERING FOR FREQUENCY SELECTION



ELECTRICAL APPLICATION OF VIBRATIONS AND WAVES

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

In electronic instrumentation, the detection of signals (currents and voltages) of a given frequency or range of frequencies often is desirable. The elimination of undesired signals and the selection of desirable ones is called "electronic filtering" and is accomplished for frequency selection by means of reactive circuits. A reactive circuit has capacitance and inductance and is so named because capacitors and inductors react to changes in the amplitude of a signal. Frequency dependence of capacitive and inductive reactance permits their use in filtering for frequency selection.

In this module, the student will be introduced to six basic filters: high-pass, low-pass, selective, notch, band-pass, and band stop. Principles of operation and design equations are presented, together with appropriate examples. In the laboratory, the student will measure the output signal, as a function of frequency, for various filters.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves"; and Application Modules IIE1, "Alternating Voltages and Current," and IIE2, "Phase Relationships in a.c. Circuits."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Accomplish the following for low- and high-pass filters:

  Draw a schematic and explain the operating principle.
  Evaluate the design equation and find the cutoff frequency.
  Choose R, L, and C for desired ratio of input to output voltage V_{out}/V_{in} and frequency in practical applications.

- Perform the following for tuned circuit filters:

  Draw a simple schematic.
  Explain the operating principle.
  Find the resonant frequency.

- Given the appropriate equipment, measure the output signal, as a function of frequency, for various filters.

FREQUENCY AND CAPACITIVE REACTANCE

The opposition of a capacitor to alternating current is called "capacitive reactance." The mathematical relationship between capacitive reactance, frequency, and capacitance is indicated by Equation 1.

$$X_C = \frac{1}{2\pi fC} \qquad (1)$$

where

X_C = capacitive reactance.
f = frequency.
C = capacitance.

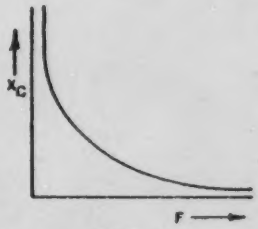


FIG. 1

Figure 1 shows the frequency dependence of X_C graphically. Since capacitive reactance is large for currents of low frequency and small for those of high frequency, a capacitor can be used to separate currents of low and high frequency in an alternating signal of mixed frequencies.

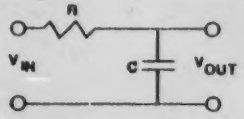


FIG. 2

A low-pass filter is illustrated in Figure 2. The resistance R is not frequency dependent, but is constant for all frequencies, as indicated by Figure 3. The total impedance Z of the RC circuit is found vectorially. Since the voltage V_C across the capacitor is 90° out of phase with the current, it also is 90° out of phase with the voltage across R. Impedance is given by Equation 2 and plotted in Figure 4.

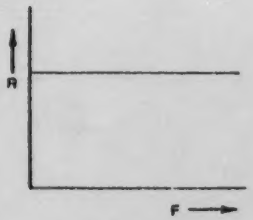


FIG. 3

$$Z = \sqrt{R^2 + X_C^2} \qquad (2)$$

where

Z = total impedance.
R = resistance.

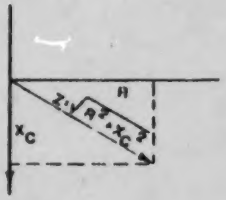


FIG. 4

The input voltage V_{in} and current I are related mathematically by the equation

$$V_{in} = IZ.$$

The output voltage V_{out} is

$$V_{out} = IX_C.$$

so that the ratio V_{out}/V_{in} is as follows:

$$\frac{V_{out}}{V_{in}} = \frac{IX_C}{IZ}$$

$$= \frac{X_C}{Z} \qquad \text{(canceling I)}$$

$$= \frac{X_C}{\sqrt{R^2 + X_C^2}} \qquad \text{(substituting for Z)}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{\frac{R^2}{X_C^2} + 1}} \qquad \left(\text{multiplying by } \frac{X_C^{-1}}{X_C^{-1}}\right)$$

By use of the relation between X_C and f, Equation 3 becomes the design equation for the low-pass filter.

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + (2\pi fRC)^2}} \qquad (3)$$

where

V_{in} = input voltage.
V_{out} = output voltage.
f = frequency of signal.
R = resistance.

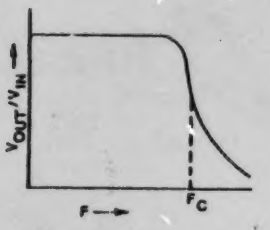


FIG. 5

For high-frequency signals, the ratio V_{out}/V_{in} is small; for low frequency, the ratio is nearly one. The voltage out, therefore, is partial to low frequency components, and the device functions as a low-pass filter. This result is shown graphically in Figure 5.

If (instead) the output voltage is taken across the resistor, the RC series circuit (shown in Figure 6) becomes a high-pass filter. The design equation for $V_{out} = IR$ is Equation 4.

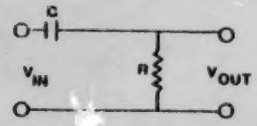


FIG. 6

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + (2\pi fRC)^{-2}}} \qquad (4)$$

Equation 4 is employed to solve a problem in Example A.

EXAMPLE A: USE OF RC DESIGN EQUATION FOR HIGH-PASS FILTER

Given: An RC high-pass filter has $R = 600\ \Omega$ and $C = 1 \times 10^{-6}$ f. The frequency of an undesirable signal is 60 Hz.

Find: The ratio V_{out}/V_{in} for the undesirable 60 Hz component.

Solution:

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + (2\pi fRC)^{-2}}} \qquad \text{(Equation 4)}$$

$$= \frac{1}{\sqrt{1 + (2\pi \times 60\ \text{Hz} \times 600\ \Omega \times 1 \times 10^{-6}\text{f})^{-2}}}$$

$$\frac{V_{out}}{V_{in}} = 0.22$$

High-frequency signals are passed by the filter, and low frequencies are stopped. The high-pass filter is V_{out}/V_{in} and dependent upon frequency, as shown graphically in Figure 7.

When the frequency f is such that $X_C = R$, the ratio V_{out}/V_{in} is $1/\sqrt{2}$, or approximately 0.707. The frequency defined by the condition $X_C = R$ is called the "cutoff frequency" f_c and is shown in both Figure 5 and Figure 7. It is related mathematically to the resistance R and capacitance, as shown by Equation 5.

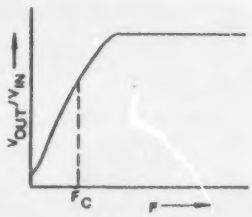


FIG. 7

$$X_C = R$$

$$\frac{1}{2\pi f_c C} = R \qquad \text{(substituting for } X_C\text{)}$$

$$f_c = \frac{1}{2\pi RC} \qquad \text{(solving for f)} \qquad (5)$$

where

f_c = cutoff frequency.

The low-pass filter removes 70.7% or more of signals having frequencies equal to, or greater than, the cutoff frequency. The high-pass filter performs likewise in removing low-frequency signals. Example B illustrates the use of Equation 5 to solve a typical problem.

EXAMPLE 8: RC LOW-PASS FILTER

Given: An RC low-pass filter has R = 10,000 Ω and C = 0.015 x 10^{-6} f.

Find: The upper cutoff frequency.

Solution:

$$f_c = \frac{1}{2\pi RC}$$

$$= \frac{1}{(2\pi \times 10{,}000\ \Omega \times 0.015 \times 10^{-6} f)}$$

$$f_c = 1060\ \text{Hz}$$

FREQUENCY AND INDUCTIVE REACTANCE

Series RL circuits also can be used as high- and low-pass filters. The opposition to alternating current by inductance is the inductive reactance given by Equation 6.

$$X_L = 2\pi fL \qquad (6)$$

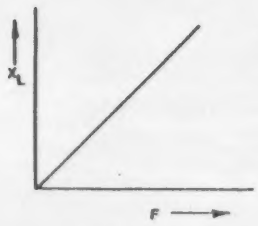


FIG. 8

where

X_L = inductive reactance.
L = inductance.
f = frequency.

This relationship is shown in Figure 8. Since voltage V_L and current are 90° out of phase, the total impedance of the RL series circuit is found vectorially, as shown in Figure 9 and in Equation 7.

$$Z = \sqrt{R^2 + X_L^2} \qquad (7)$$

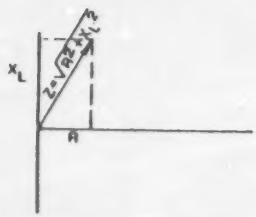


FIG. 9

where

Z = total impedance.

For the low-pass filter of Figure 10, the voltage V_{out} is IR; and the design equation is Equation 8.

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + \left(\frac{2\pi fL}{R}\right)^2}} \qquad (8)$$

As for the low-pass RC filter, V_{out}/V_{in} is small for high-frequency signals and nearly one for low. The RL high-pass filter of Figure 11 reverses the roles of resistance and inductance and yields the design equation given in Equation 9.

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + \left(\frac{R}{2\pi fL}\right)^2}} \qquad (9)$$

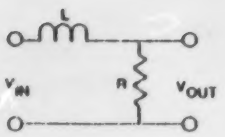


FIG. 10

The cutoff frequency now is defined by the condition $X_L = R$, for which V_{out}/V_{in} is approximately 0.707 and is given by Equation 10.

$$f_c = \frac{R}{2\pi L} \qquad (10)$$

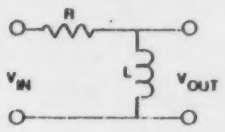


FIG. 11

Example C illustrates RL high- and low-pass filters.

EXAMPLE C: RL FILTERS

Given: An RL filter has $L = 25 \times 10^{-3}$ H, and $R = 2000\ \Omega$.

Find: The cutoff frequency.

Solution:

$$f_c = \frac{R}{2\pi L}$$

$$= \frac{2000\ \Omega}{(2\pi)(25 \times 10^{-3}\ \text{H})}$$

$$f_c = 12{,}730\ \text{Hz}$$

If the circuit is used as a high-pass filter, 70.7% of better of signals having $f \geq 12{,}730$ Hz will be passed; similarly, the circuit will pass 70.7% of signals having $f \leq 12{,}730$ Hz if used as a low-pass filter.

TUNED CIRCUIT FILTERS

Resonant RLC series circuits are used in filters with the purpose of passing or stopping signals in a particular range of frequencies. Such a circuit is said to be tuned to the frequency at the center of the range or band. The band may be broad, or it

may be so narrow that it permits only a "single" frequency within a few Hertz.

Equation 11 gives the total impedance of a resonant RLC series circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad (11)$$

This total impedance is shown vectorially in Figure 12. At resonance, the impedance of the series circuit is minimum and, therefore, the current I is maximum. Equation 11 indicates minimum impedance when $X_L = X_C$, for which $Z = R$. This resonant frequency is derived from Equation 12 as follows:

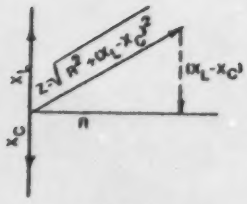


FIG. 12

$$X_L = X_C$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \qquad (12)$$

where

f_r = resonant frequency.

Also, at resonance, since $X_L = X_C$, then $V_L = V_C$, and the total voltage drop $(V_L + V_C)$ across the inductor and capacitor is zero for a resistanceless inductor.

The voltage V_L across the inductor is the following:

$$V_L = IX_L$$

$$= \frac{V_{in}X_L}{Z} \qquad \left(\text{since } \quad = \frac{V_{in}}{Z}\right)$$

$$V_L = \frac{V_{in}X_L}{R} \qquad (\text{since } Z = R \text{ at } f)$$

If $X_L > R$, then the voltage V_L across the inductor actually is larger than the total input voltage V_{in}. A large, single-frequency (resonant frequency) potential is available across either the capacitor or inductor if the resistance R is small. The RLC series circuit used in this manner is called a "selective frequency filter." Usually, no resistance is added to the LC

series, and R then is the resistance of the inductor. A schematic of the selective frequency filter is shown in Figure 13.

The ratio V_{out}/V_{in} is shown graphically as a function of frequency in Figure 14. At resonance, the quality factor Q of the inductor or circuit is defined by Equation 13, and is found in the following way:

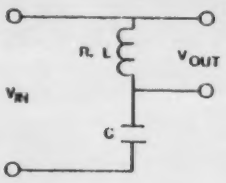


FIG. 13

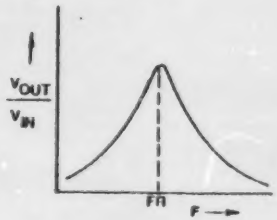


FIG. 14

$$\frac{V_{out}}{V_{in}} = \frac{V_L}{V_{in}}$$

$$= \frac{X_L}{R} \qquad \left(\text{since } V_L = \frac{V_{in} X_L}{R}\right)$$

$$= \frac{2\pi f_r L}{R} \qquad (\text{since } X_L = 2\pi f_r L)$$

$$= \frac{2\pi L}{2\pi R\sqrt{LC}} \qquad \left(\text{since } f_r = \frac{1}{2\pi\sqrt{LC}}\right)$$

$$\frac{V_{out}}{V_{in}} = \sqrt{\frac{L}{R^2 C}}$$

or $$Q = \sqrt{\frac{L}{R^2 C}}$$

Large Q-values are characteristic of sharply-tuned filters. Typical Q-values lie between 10 and 100. Equations 12 and 13 are used to solve a problem in Example D.

EXAMPLE D: SELECTIVE FREQUENCY FILTER

Given: A selective frequency filter has $L = 25 \times 10^{-3}$H and $C = 0.001 \times 10^{-6}$F. The resistance of the inductor is 50 Ω.

Find: (a) The resonant frequency.
(b) The Q-factor.

Solution: (a) $$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{(2\pi \times \sqrt{(25 \times 10^{-3}\text{H})(0.001 \times 10^{-6}\text{F})}}$$

$$f_r = 31{,}800 \text{ Hz}$$

(b) $Q = \sqrt{\frac{L}{R^2 C}}$

$$= \sqrt{\frac{25 \times 10^{-3} H}{(50\ \Omega^2)(0.001 \times 10^{-6} F)}}$$

$$Q = 100$$

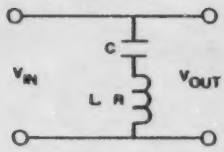


FIG. 15

At resonance, the voltage ($V_L + V_C$) across the LC series is due only to the resistance R of the inductor coil and is very small compared to the input voltage V_{in} for a circuit with a high Q. The notch filter shown in Figure 15 is an RLC resonant circuit in which at resonance the resistance R is due only to the inductor coil (Equation 14).

$$V_{out} = V_L + V_C$$

$$V_{out} = IR \qquad (14)$$

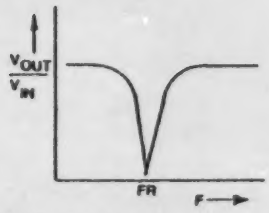
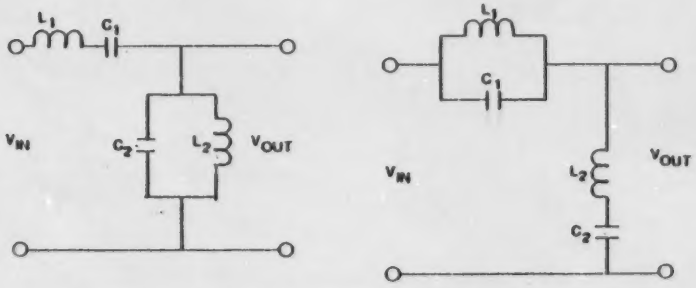


FIG. 16

The resulting output V_{out} has effectively no signals of frequency f_r. The notch filter's V_{out}/V_{in} ratio is shown graphically as a function of frequency f in Figure 16.

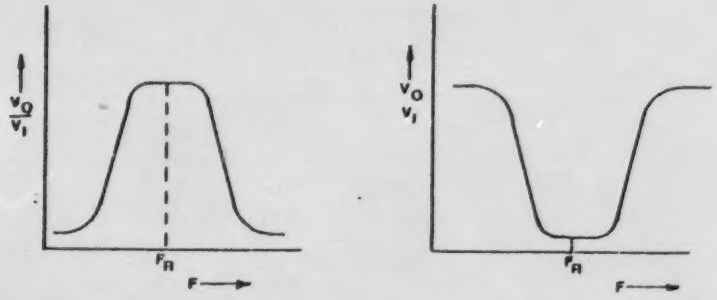
Band-pass and band-stop filters are used when the necessity arises to pass or block signals with a particular range of frequencies. They are combinations of LC series and parallel circuits tuned to the center of the band. Simple band-pass and band-stop schematics are shown in Figure 17a and 17b, respectively.



(a) BAND PASS (b) BAND STOP

FIG. 17

The impedance of an RLC parallel circuit is maximum at resonance, in contrast to the RLC series circuit. The parallel circuit of the band-pass filter is, effectively, an open circuit to signals with frequencies in the band; i.e., those near the resonant frequency $f_r = 1/2\pi\sqrt{LC}$, and the series circuit acts as a short to the same signals. Signals having frequency within the band, therefore, are passed by the series circuit and appear across the parallel circuit as V_o. Those outside the band (far away from resonance) are stopped by the series circuit as if it were open and are passed by the parallel circuit as if it were a short circuit and, thereby, are removed from V_o. A similar analysis applies to the band-stop filter, which removes from V_o signals within the band. These characteristics are shown graphically in Figure 18.



(a) BAND PASS (b) BAND STOP

FIG. 18

SUMMARY

The elimination of undesired signals and the selection of desirable ones is called "electronic filtering" and is accomplished for frequency selection by means of reactive circuits. A reactive circuit has capacitance and inductance and is so named because capacitors and inductors react to changes in the amplitude of a signal. Frequency dependence of capacitive and inductive reactance permits their use in filtering for frequency selection. Low-pass filters are employed in circuits designed to pass only the lower frequencies and to attenuate any frequencies above a selected cut-off frequency. High-pass filters block signals when the reactance of the capacitance becomes large enough at very low frequencies. Resonant RLC series circuits are used in filters to stop signals

in a particular range of frequencies. Such filters include the band-pass and band-stop types.

HIGH- AND LOW-PASS FILTERS

MATERIALS

Signal generator.
Oscilloscope.
0.1 μf capacitor.
25 mH inductor.
1 KΩ resistor.
Ohmmeter.

PROCEDURE

1. Build the RC series circuit illustrated in Figure 19.

FIG. 19

2. Measure the input voltage $V_{in} = V_{ac}$ (peak-to peak) with oscilloscope, and record value in Data Table 1. (Record signal-generator setting for V_{in} and use same setting in Step 3 of second experiment.)

3. Calculate cutoff frequency f_c, and record results in Data Table 1.

4. For the high-pass filter, $V_{out} = V_{ab}$. Measure V_{out} (peak-to-peak) as a function of frequency for high-pass filter, taking care to record more points near cutoff frequency. Include V_{out} for f_c as one of your points.

5. Divide each V_{out} by V_{in}, and record in Data Table 1. Plot ratio V_{out}/V_{in} (vertical axis) as a function of log of f (horizontal axis). Compare value of V_{out}/V_{in} at f_c to 0.707.

6. Repeat Steps 4 and 5 for low-pass filter for which $V_o = V_{bc}$.

7. (Optional.) Replace capacitor with inductor, and repeat Steps 1-6 for RL series filters. Note that for the high-pass filter, $V_{out} = V_{bc}$, and for the low-pass, $V_{out} = V_{ab}$.

8. Explain the principles demonstrated by this experiment.

TUNED-CIRCUIT FILTERS

MATERIALS

Signal generator.
Oscilloscope.
0.01 μf capacitor.
25 mH inductor.
1 KΩ resistor.
Ohmmeter.

PROCEDURE

1. Build LC series circuit shown in Figure 20. Measure resistance R of inductor, and record value in Data Table 2.

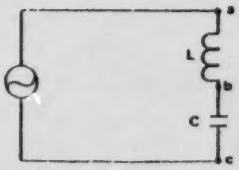


FIG. 20

2. For the selective frequency filter, observe output signal $V_{out} = V_{ab}$ (or $- V_{bc}$) with oscilloscope as frequency changes. At resonance, output signal V_{out} is maximum. Measure frequency f_r and V_{out} at resonance, and record in Data Table 2.

3. Calculate the quality factor $Q = V_{out}/V_{in}$ at resonance, using the measured value of V_{in} from Step 2 of first experiment. (V_{in} was recorded in Data Table 1.) Compare your experimental value with that calculated from the equation $Q = \sqrt{L/R^2C}$.

4. For notch filter, observe output signal $V_{out} = V_{ac}$ with oscilloscope as frequency changes. At resonance, this output signal is minimum. Measure resonant frequency f_r, and record in Data Table 2.

5. Calculate resonant frequency from the equation $f_r = 1/2\pi\sqrt{LC}$, and record in Data Table 2. Compare with values measured in Steps 2 and 4. To what do you attribute any discrepancies?

DATA TABLE 1					
V_{in} = _____ volts			f_c = _____ Hz		
Signal Generator Setting for V_{in} _____.					
High-pass Filter			Low-pass Filter		
f (Hz)	V_{out} (volts)	$\frac{V_{out}}{V_{in}}$	f (Hz)	V_{out} (volts)	$\frac{V_{out}}{V_{in}}$

DATA TABLE 2

R = _____ ohms

Selective Frequency Filter:

f_r = _____ Hz

V_{out} = _____ volts

Notch Filter:

f_r = _____ Hz

RESULTS

$Q = \frac{V_{out}}{V_{in}}$ = _____

$Q = \sqrt{\frac{L}{R^2C}}$ = _____

$f_r = \frac{1}{2\pi\sqrt{LC}}$ = _____

SELF CHECK

1. Draw a schematic of, and explain the operating principle for, the following filters:

   a. Low-pass (RC and RL).
   b. High-pass (RC and RL).
   c. Selective.
   d. Notch.
   e. Band pass.
   f. Band stop.

2. A low-pass filter has R = 5000 Ω and C = 0.16 μf.

   a. Find the cutoff frequency.
   b. What inductance L is needed for an RL low-pass filter with the same resistance and cutoff frequency?

3. Find V_{out}/V_{in} at f = 1 MHz for a high-pass filter having the following values:

   a. R = 5000 Ω and C = 320 pf.
   b. R = 500 Ω and L = 8.0 mH.

4. Find the following for a selective-frequency filter having R = 1 Ω, L = 250 μH, and C = 0.1 μf:

   a. The resonant frequency.
   b. The quality factor.

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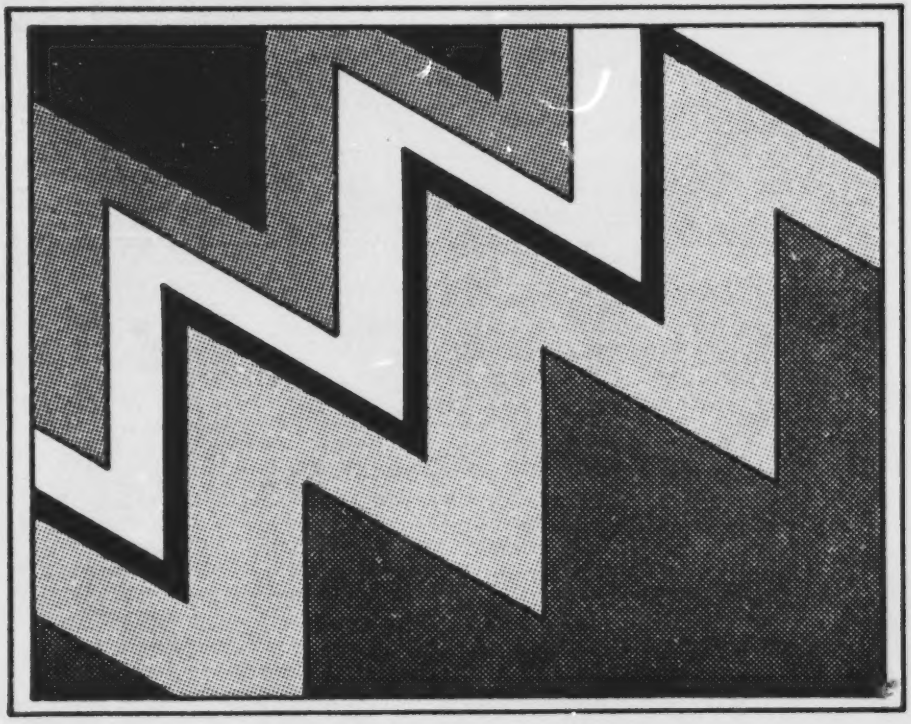
UTC
UNIFIED TECHNICAL CONCEPTS

MODULE 11 E 7

MODULATION OF ELECTRICAL WAVES



ELECTRICAL APPLICATION

OF VIBRATIONS AND WAVES

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

One of the most important applications of waves is the transmission of information, in the form of varying electromagnetic waves, through space. Such transmission is the basis of all modern communication. The process by which information is introduced onto a transmitted "carrier" wave is called "modulation." This module discusses the two basic types of modulation used in modern communication systems — amplitude modulation (AM) and frequency modulation (FM). In the laboratory, the student will produce and observe the form of AM waves.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following terms:

  Carrier wave.
  Modulation.
  Amplitude modulation.
  Frequency modulation.
  Phase modulation.

- Sketch the wave forms produced by the following modulation techniques:

  Amplitude modulation.
  Frequency modulation.

- Describe the formation of sidebands when a wave is modulated, and define the terms USB and LSB.

- Given the frequency of a carrier wave and the modulation frequency, determine the frequencies of the sidebands produced.

- Explain, with the use of a diagram, the percent modulation of an AM wave.
- Given the appropriate equipment, produce and observe AM waves.

DISCUSSION

The purpose of a communications system is to transmit "intelligence" or "information" from one point to another. We usually think of intelligence as being speech or music, but it is not limited to that. Computers, for example, "speak" in a coded language in which the absence or presence of pulses is translated into "words," much like the Morse Code. Telemetry systems carry information about an astronaut's heartbeat or respiratory system; they transmit this information from the space ship to an earth station without a "word" being spoken.

If speech is used as an example of intelligence, everybody can visualize how this intelligence is transmitted from one person to another within earshot of the first. But what if the second person were a hundred miles away? Audio waves do not travel (propagate) that far, but radio waves (depending upon frequency) do. To accomplish communication, therefore, the intelligence is "modulated" onto an electromagnetic wave of radio frequency, which then carries the information to its destination (like a bus carrying a passenger). "Modulation" is the process of altering a radio frequency in such a manner that the alteration is proportional to the intelligence. At the receiving point, "demodulation" occurs when the intelligence is separated from the radio frequency carrier.

Despite some of the many phrases (such as "two-tone modulation," "frequency-shift keying," "pulse-duration modulation," "delta modulation," etc.), used in communications vocabulary today, there are only two methods of modulating a radio-frequency (carrier) wave. An rf carrier wave is nothing more than a sine wave that travels through space. This sine wave is an electrical signal whose value, e, at any given instant is given by Equation 1.

$$e = E \sin (2\pi ft + \theta) \qquad (1)$$

where

E = peak amplitude (electric field).
f = carrier frequency in Hertz.
θ = phase angle displacement.

This carrier wave (Figure 1), in and of itself, carries no intelligence, except in that it is either present or absent. Control of the presence or absence of a carrier wave according to a pre-arranged formula is the basis for Morse Code, which is aside from the discussion of modulation presented here.

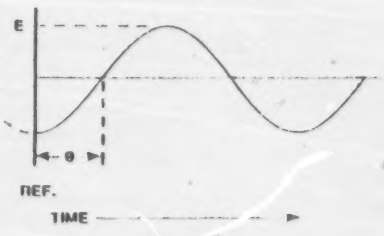
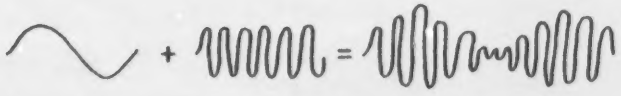


FIG. 1

The two methods for the modulation of a carrier wave are called "Amplitude Modulation" and "Angular Modulation" (which consists of Frequency Modulation and Phase Modulation).

In an amplitude modulation system (AM), the maximum amplitude, E, is varied at the intelligence rate. The transmitted waveform is depicted in Figure 2.



1 KHz MODULATED ONTO 1 MHz PRODUCES AM WAVEFORM

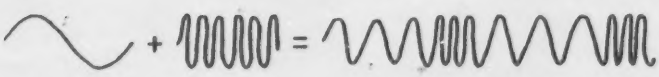
FIG. 2

In an Angular Modulation system (Frequency Modulation or Phase Modulation), the frequency, f, or phase angle, θ, is varied at the intelligence rate. The reason that the two methods, FM or PM, are grouped together is that there is very little difference between them. The subtle differences are beyond the scope of this module, but the reason why they are similar is something that is within the realm of this discussion.

A wave with constant frequency always will start a new cycle at the same reference point relative to the last cycle; so one can say that its phase angle is "constant." If, however, the wave frequency were increased, the starting point of the next cycle would be earlier, which would appear to be a shift in phase angle. If the frequency were decreased, the phase angle would shift in the other direction; thus, a variation in frequency produces a variation in phase angle. Similarly, if the phase angle is directly varied (modulated), the frequency of the carrier increases or decreases. Thus, a carrier cannot be frequency modulated without also producing phase-modulation, and a carrier cannot be phase modulated without also producing frequency-modulation.

Of the two methods, FM is the more popular because at the receiving end a demodulator that responds to frequency change rather than to a phase change can be built. An FM demodulating circuit is called a "discriminator," and circuits such as the Foster-Seeley discriminator provide a good linear recovery of the intelligence of a frequency-modulated waveform. A new circuit, which responds to phase changes and which is referred to as the "phase-locked loop" (PLL), is becoming increasingly popular as IC technology advances; and perhaps someday the emphasis of angular modulation will shift from FM to PM.

In an FM system, the transmitted waveform takes the shape illustrated in Figure 3.



1 KHz MODULATED ONTO 1 MHz PRODUCES FM WAVEFORM

FIG. 3

Notice that the FM waveform has a constant amplitude but a variable frequency that varies at the intelligence rate; whereas, the AM waveform has a constant frequency

with a variable amplitude that varies at the intelligence rate. In FM parlance, the carrier frequency is said to be "deviated" from its center frequency, where the amount of deviation (how far away from the center frequency) is proportional to the amplitude of the intelligence and where the rate of deviation (how fast the deviation occurs above and below the center frequency) is proportional to the frequency of the intelligence.

Regardless of whether AM or FM, the modulation process produces "sidebands." Sidebands are radio frequencies above and below the carrier frequency, and are known as "upper sidebands" (USB) or "lower sidebands" (LSB). The reason for the existence of sidebands is a mathematical phenomena. Fourier, a French mathematician, proved that all non-sinusoidal waves can be constructed from sine waves. For instance, the mathematical formula for a sawtooth waveform is —

$$e = -2(\sin 2\pi ft + 1/2\sin 2x2\pi ft + 1/3\sin 3x2\pi ft \ldots)$$

A sawtooth wave has the form shown in Figure 4.

FIG. 4

The above formula might look horribly complex and tend to scare one off, but observe it closely for a moment. The first term, sin 2πft, is what is called the "fundamental frequency," or simply a sine wave of frequency f. Notice that the second term is "2x2πf"; therefore, it is twice the frequency of the fundamental, known as the "second harmonic." Its amplitude is 1/2 the amplitude of the fundamental. The third term is a sine wave whose frequency is 3 times the fundamental (3rd harmonic) and whose peak amplitude is 1/3 the amplitude of the fundamental. What Fourier is saying, in plain English, is that if a sine wave of some frequency, f, is added to another sine wave of frequency 2xf at 1/2 the amplitude, etc., the resultant waveform would be a sawtooth wave. The important part of this analysis is that the converse of this statement is true. If a sawtooth wave were the desired "intelligence" to be modulated onto a carrier, the transmission system would have to transmit all the individual sine waves that make up that sawtooth process. Then, if all the sine waves could be added appropriately together at the receiver, the sawtooth waveform would be recovered.

The Fourier analysis is the underlying principle behind the invention of the Hammond organ. A true pipe organ, in blowing air across the reed into a tuned pipe, produces a sawtooth wave. (The reed slowly flexes with air pressure to the point at which it reaches its mechanical limit. Then it quickly snaps back to its starting position, hence tracing out a sawtooth pattern.)

The Hammond organ electronically recreates this sound by adding sine waves together according to the above formula. The player controls the "amplitude" of each "harmonic" by manipulating a "draw bar." The right combination of sine wave amplitudes and frequencies produces a "sawtooth," or "pipe organ," sound.

Only sine waves propagate; therefore, any non-sinusoidal wave, such as a triangular wave or an AM or FM waveform, does not travel as such, but travels as its individual sine-wave counterparts. An AM waveform does not travel as a single sine wave of carrier frequency whose amplitude is varying at the intelligence rate, but actually travels as three individual sine waves (a carrier, an USB, and a LSB), which, when added together, have the appearance of an AM waveform.

Let us look at a specific example to clarify this. Assume a carrier frequency, f_c, of 100 KHz, and an intelligence frequency, f_I, of 1 KHz. Adding these two frequencies together in a linear device does not produce modulation. The output would simply be the two frequencies, 100 KHz and 1 KHz. The 100-KHz frequency, being in the rf range, would propagate, but the 1-KHz one, being an audio frequency, would not. If the two frequencies were added, or mixed, in a non-linear device, however, modulation would occur. The output frequencies would be the two originals plus their sums and differences. Thus, with 1 KHz and 100 KHz going into the mixer, the output contains 1 KHz, 100 KHz, 101 KHz, and 99 KHz. The 100-KHz frequency is the carrier; the 99-KHz frequency, the LSB; and the 101-KHz frequency, the USB. These three frequencies are transmitted, but the fourth (1 KHz) is rejected. The three frequencies when added together produce a waveform that looks like an AM waveform. Let us now look at the mathematical formula for an AM waveform:

$$e = E_{sin}(2\pi f_c t) - \frac{Em^2}{2}\cos(2\pi f_c t + 2\pi f_I t) + \frac{Em^2}{2}\cos(2\pi f_c t - 2\pi f_I t)$$

or

$$e = \underbrace{E_{sin}[2\pi(100KHz)t]}_{\text{carrier}} - \underbrace{\frac{Em^2}{2}\cos\,2\pi[101KHz]t}_{\text{USB}} + \underbrace{\frac{Em^2}{2}\cos\,2\pi[99KHz]t}_{\text{LSB}}$$

Here, "m" is the modulation index, a number from 0 to 1, which represents how strong a carrier is modulated by the intelligence. Notice that if m = 0 (no modulation), the second and third term in the equation would also be 0, leaving only the carrier to be transmitted. When m = 1 (maximum modulation), the amplitude of the second term (USB) and of the third term (LSB) each would be 1/2 the carrier amplitude.

To summarize the example, if a 100-KHz carrier were fully modulated (100%) by a 1-KHz intelligence frequency, three sine waves would be produced: (1) a carrier frequency of 100 KHz and amplitude of E, (2) a lower-sideband frequency of 99 KHz and amplitude of E/2, and (3) an upper-sideband frequency of 101 KHz and amplitude of E/2. These three frequencies, when traveling together, are the AM waveforms depicted in Figure 5.

The significance of the above statement is this: If the receiver recovered only the 100-KHz carrier and had such sharp selectivity that it rejected the 99- and 101-KHz sidebands, the receiver would recover no intelligence, since the carrier, of and by itself, is only a single sine wave and is not an AM waveform. To recover the intelligence and to reconstruct the AM waveform, all three frequencies must be received. The term "bandwidth" is a description of how wide (in frequency) a communications channel is. In the above example, the bandwidth of the channel needed to transmit and recover the AM waveform is 2 KHz (101-99 KHz). If the intelligence frequency in the above example were changed to 5 KHz, then the USB would be 105 KHz; the LSB would be 95 KHz; and the bandwidth requirement would be 10 KHz. In the AM broadcast band, the spacing between channels (and, therefore, the maximum bandwidth) is 10 KHz; consequently, the highest intelligence frequency that can be modulated onto the carrier is 5 KHz.

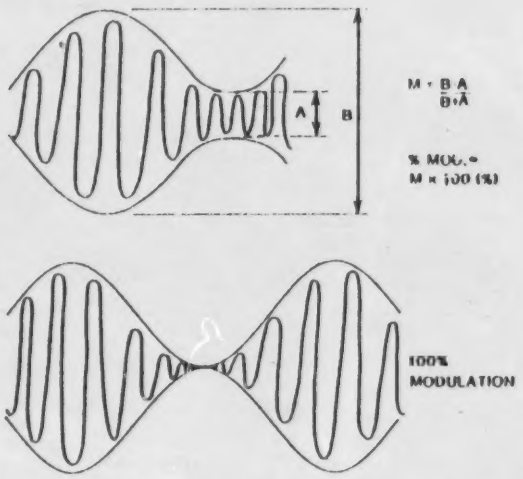


FIG. 5

The percentage of modulation for any AM waveform can be found from Equation 2.

$$m = \frac{B - A}{B + A} \qquad (2)$$

where

B = maximum amplitude of modulated wave.
A = minimum amplitude of modulated wave.

A value of 100% modulation is achieved when the peaks just close. Note that when this happens, "A" = 0, and m = B - 0/B + 0, or "1," and % modulation = 100%.

When the modulation index is known, the total power, P_t, can be calculated from Equation 3.

$$P_t = P_c\left(1 + \frac{m^2}{2}\right) \qquad (3)$$

Thus, if a 100-watt carrier were modulated 100% by an intelligence frequency, the total radiated power would be —

$$100(1 + 1/2) = 100(1.5) = 150 \text{ watts.}$$

If you have access to a computer or microprocessor that speaks "BASIC," here is a simple, short program that will demonstrate that an AM waveform is composed of individual sine waves added together. The program uses the general formula for an AM waveform and plots the instantaneous voltage for many samples of time. The result: connect all the "X's" and you have an AM waveform.

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10 FOR T = 0 TO 100
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30 Let e = e - 8*cos(1.6493361*T)
40 Let e = e + 8*cos(1.4922565*T)
50 Let e = INT(e + 40)
60 PRINT T; tab(e); "X"
70 NEXT T
80 END
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SUMMARY

Information is transmitted through space in the form of electromagnetic waves. Amplitude modulation is accomplished by variation of the amplitude, or "strength," of a carrier wave at the frequency of the information to be transferred. Both frequency modulation and phase modulation produce a modulated wave of constant amplitude but with variations of frequency at the information rate. In all cases, the transmitted wave is not a single frequency but a range of frequencies added in the receiver to reproduce the information.

MODULATION OF ELECTRICAL WAVES

MATERIALS

RF signal generator with AM modulation. (Sargent-Welch #S-68745 or equivalent).
Oscilloscope.
Audio frequency signal generator.
AM radio.
One-meter length of insulated copper wire.

PROCEDURE

1. Connect equipment as illustrated in Figure 6.

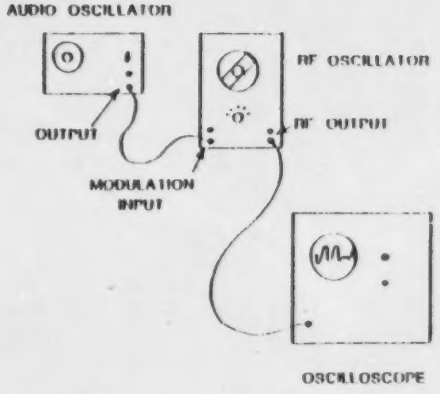


FIG. 6

2. Turn on oscilloscope and RF oscillator; set oscillator modulation to zero, output frequency to 600 KHz, and modulation to "external."

3. Set oscilloscope time base to 10 ms/cm, and obtain a stable trace. (Change time base as necessary throughout the experiment.) Sketch unmodulated wave in Data Table.

4. Turn on audio oscillator, and set output frequency to 1000 Hz. Set output level to zero. Set modulation level of RF oscillator to maximum. Set oscilloscope time base to 0.1 ms/cm.

5. Slowly increase output of audio oscillator while watching oscilloscope display. Set audio oscillator output to produce maximum modulation. Sketch amplitude modulated wave in Data Table.

6. Vary audio frequency. Observe modulated wave at several settings of oscilloscope time base, and sketch in Data Table.

7. Set audio oscillator frequency to 1000 Hz. Vary RF frequency and observe modulated wave at several settings of oscilloscope time base. Sketch in Data Table.

8. Turn off RF oscillator, and disconnect from oscilloscope. Connect insulated wire to RF output, and place near antenna of AM radio.

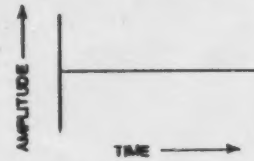
9. Turn on RF oscillator and radio. Tune radio and RF oscillator to 600 KHz, and obtain 1000 Hz tone from radio.

10. Vary audio frequency as before, and observe tone from radio.

11. Answer questions in Data Table.

DATA TABLE

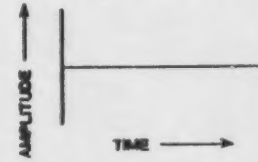
1. Sketch unmodulated RF wave observed in Step 3.



RF frequency = _____

Oscilloscope time base = _____

2. Sketch modulated wave observed in Step 5.



RF frequency = _____

Audio frequency = _____

Oscilloscope time base = _____

3. Sketch two waveforms observed as audio frequency changed in Step 6.

RF frequency = _____ RF frequency = _____

Audio frequency = _____ Audio frequency = _____

Describe how variation of the audio frequency changes the modulated waveform.

4. Sketch two waveforms observed as RF frequency changes in Step 7.

RF frequency = _____ RF frequency = _____

Audio frequency = _____ Audio frequency = _____

Describe how variation of the RF frequency changes the modulated waveform.

5. With an audio frequency of 1000 Hz and an RF frequency of 600 KHz, what frequencies are present in the modulated output?

SELF CHECK

1. Define the following terms:

   a. Carrier wave.
   b. Modulation.
   c. AM.
   d. FM.
   e. PM.

2. Sketch an amplitude-modulated waveform.

3. Sketch a frequency-modulated waveform.

4. Describe the relationship of FM to PM.

5. Describe the formation of sidebands when a wave is modulated.

6. A carrier frequency of 200 KHz is modulated at a frequency of 20 KHz. What are the frequencies of the upper and lower sidebands?

7. Define "percent modulation of an AM wave" with the use of a diagram.

REFERENCES

Greenberg, Leonard H. Physics with Modern Applications. Philadelphia: W. B. Saunders Company, 1978.

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The Way Things Work. Vol. 1. New York: Simon and Schuster, 1967.

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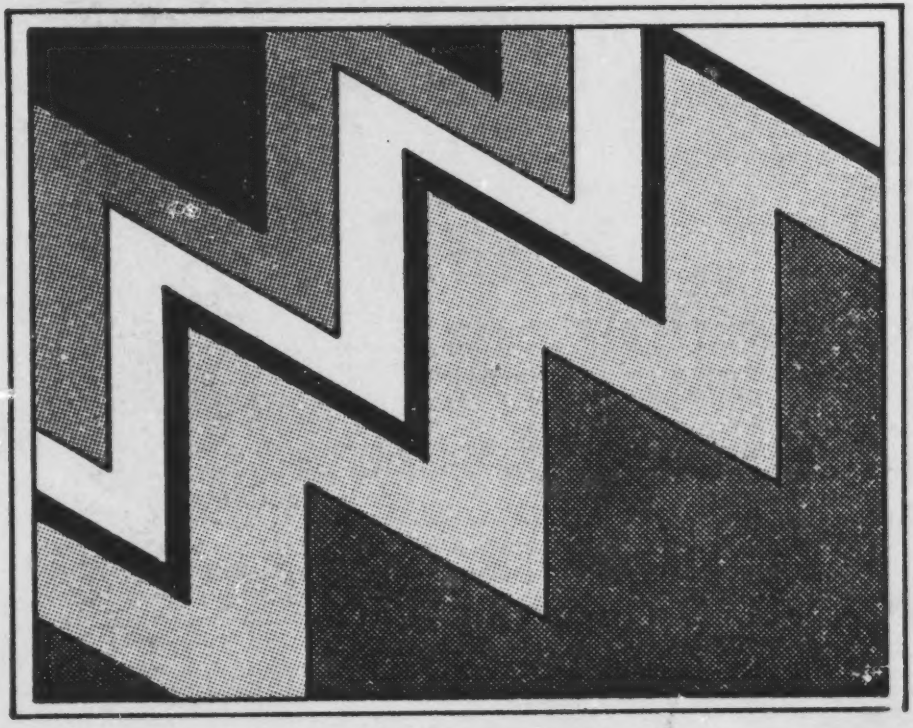
UNIFIED TECHNICAL CONCEPTS

MODULE 11 E 7

MODULATION OF ELECTRICAL WAVES



ELECTRICAL APPLICATION

OF VIBRATIONS AND WAVES

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

One of the most important applications of waves is the transmission of information, in the form of varying electromagnetic waves, through space. Such transmission is the basis of all modern communication. The process by which information is introduced onto a transmitted "carrier" wave is called "modulation." This module discusses the two basic types of modulation used in modern communication systems — amplitude modulation (AM) and frequency modulation (FM). In the laboratory, the student will produce and observe the form of AM waves.

PREREQUISITES

Student should have completed Concept Module II-0, "Vibrations and Waves."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following terms:

  Carrier wave.
  Modulation.
  Amplitude modulation.
  Frequency modulation.
  Phase modulation.

- Sketch the wave forms produced by the following modulation techniques:

  Amplitude modulation.
  Frequency modulation.

- Describe the formation of sidebands when a wave is modulated, and define the terms USB and LSB.

- Given the frequency of a carrier wave and the modulation frequency, determine the frequencies of the sidebands produced.

- Explain, with the use of a diagram, the percent modulation of an AM wave.
- Given the appropriate equipment, produce and observe AM waves.

DISCUSSION

The purpose of a communications system is to transmit "intelligence" or "information" from one point to another. We usually think of intelligence as being speech or music, but it is not limited to that. Computers, for example, "speak" in a coded language in which the absence or presence of pulses is translated into "words," much like the Morse Code. Telemetry systems carry information about an astronaut's heartbeat or respiratory system; they transmit this information from the space ship to an earth station without a "word" being spoken.

If speech is used as an example of intelligence, everybody can visualize how this intelligence is transmitted from one person to another within earshot of the first. But what if the second person were a hundred miles away? Audio waves do not travel (propagate) that far, but radio waves (depending upon frequency) do. To accomplish communication, therefore, the intelligence is "modulated" onto an electromagnetic wave of radio frequency, which then carries the information to its destination (like a bus carrying a passenger). "Modulation" is the process of altering a radio frequency in such a manner that the alteration is proportional to the intelligence. At the receiving point, "demodulation" occurs when the intelligence is separated from the radio frequency carrier.

Despite some of the many phrases (such as "two-tone modulation," "frequency-shift keying," "pulse-duration modulation," "delta modulation," etc.), used in communications vocabulary today, there are only two methods of modulating a radio-frequency (carrier) wave. An rf carrier wave is nothing more than a sine wave that travels through space. This sine wave is an electrical signal whose value, e, at any given instant is given by Equation 1.

$$e = E \sin (2\pi ft + \theta) \qquad (1)$$

where

E = peak amplitude (electric field).
f = carrier frequency in Hertz.
θ = phase angle displacement.

This carrier wave (Figure 1), in and of itself, carries no intelligence, except in that it is either present or absent. Control of the presence or absence of a carrier wave according to a pre-arranged formula is the basis for Morse Code, which is aside from the discussion of modulation presented here.

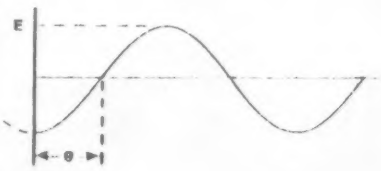
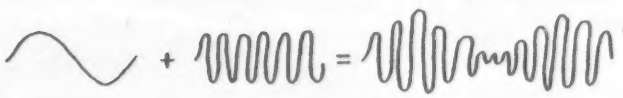


FIG. 1

The two methods for the modulation of a carrier wave are called "Amplitude Modulation" and "Angular Modulation" (which consists of Frequency Modulation and Phase Modulation).

In an amplitude modulation system (AM), the maximum amplitude, E, is varied at the intelligence rate. The transmitted waveform is depicted in Figure 2.



1 KHz MODULATED ONTO 1 MHz PRODUCES AM WAVEFORM

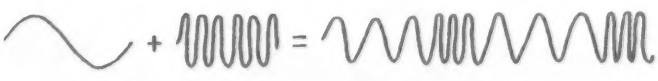
FIG. 2

In an Angular Modulation system (Frequency Modulation or Phase Modulation), the frequency, f, or phase angle, θ, is varied at the intelligence rate. The reason that the two methods, FM or PM, are grouped together is that there is very little difference between them. The subtle differences are beyond the scope of this module, but the reason why they are similar is something that is within the realm of this discussion.

A wave with constant frequency always will start a new cycle at the same reference point relative to the last cycle; so one can say that its phase angle is "constant." If, however, the wave frequency were increased, the starting point of the next cycle would be earlier, which would appear to be a shift in phase angle. If the frequency were decreased, the phase angle would shift in the other direction; thus, a variation in frequency produces a variation in phase angle. Similarly, if the phase angle is directly varied (modulated), the frequency of the carrier increases or decreases. Thus, a carrier cannot be frequency modulated without also producing phase-modulation, and a carrier cannot be phase modulated without also producing frequency-modulation.

Of the two methods, FM is the more popular because at the receiving end a demodulator that responds to frequency change rather than to a phase change can be built. An FM demodulating circuit is called a "discriminator," and circuits such as the Foster-Seeley discriminator provide a good linear recovery of the intelligence of a frequency-modulated waveform. A new circuit, which responds to phase changes and which is referred to as the "phase-locked loop" (PLL), is becoming increasingly popular as IC technology advances; and perhaps someday the emphasis of angular modulation will shift from FM to PM.

In an FM system, the transmitted waveform takes the shape illustrated in Figure 3.



1 KHz MODULATED ONTO 1 MHz PRODUCES FM WAVEFORM

FIG. 3

Notice that the FM waveform has a constant amplitude but a variable frequency that varies at the intelligence rate; whereas, the AM waveform has a constant frequency

with a variable amplitude that varies at the intelligence rate. In FM parlance, the carrier frequency is said to be "deviated" from its center frequency, where the amount of deviation (how far away from the center frequency) is proportional to the amplitude of the intelligence and where the rate of deviation (how fast the deviation occurs above and below the center frequency) is proportional to the frequency of the intelligence.

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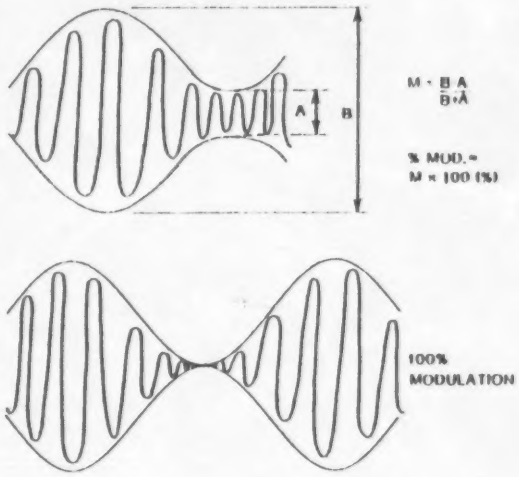

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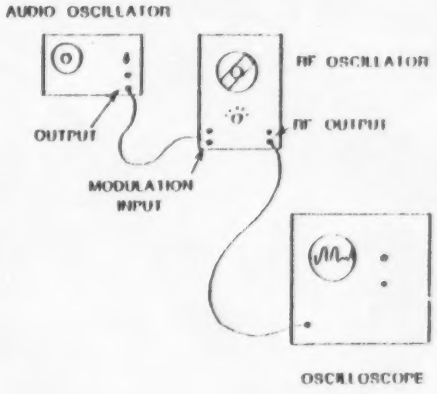

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5. Slowly increase output of audio oscillator while watching oscilloscope display. Set audio oscillator output to produce maximum modulation. Sketch amplitude modulated wave in Data Table.

6. Vary audio frequency. Observe modulated wave at several settings of oscilloscope time base, and sketch in Data Table.

7. Set audio oscillator frequency to 1000 Hz. Vary RF frequency and observe modulated wave at several settings of oscilloscope time base. Sketch in Data Table.

8. Turn off RF oscillator, and disconnect from oscilloscope. Connect insulated wire to RF output, and place near antenna of AM radio.

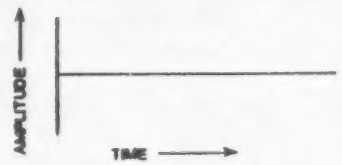
9. Turn on RF oscillator and radio. Tune radio and RF oscillator to 600 KHz, and obtain 1000 Hz tone from radio.

10. Vary audio frequency as before, and observe tone from radio.

11. Answer questions in Data Table.

DATA TABLE

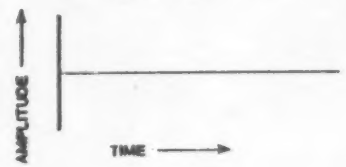
1. Sketch unmodulated RF wave observed in Step 3.



RF frequency = _____

Oscilloscope time base = _____

2. Sketch modulated wave observed in Step 5.



RF frequency = _____

Audio frequency = _____

Oscilloscope time base = _____

3. Sketch two waveforms observed as audio frequency changed in Step 6.

RF frequency = _____

Audio frequency = _____

RF frequency = _____

Audio frequency = _____

Describe how variation of the audio frequency changes the modulated waveform.

4. Sketch two waveforms observed as RF frequency changes in Step 7.

RF frequency = _____

Audio frequency = _____

RF frequency = _____

Audio frequency = _____

Describe how variation of the RF frequency changes the modulated waveform.

5. With an audio frequency of 1000 Hz and an RF frequency of 600 KHz, what frequencies are present in the modulated output?

SELF CHECK

1. Define the following terms:

   a. Carrier wave.
   b. Modulation.
   c. AM.
   d. FM.
   e. PM.

2. Sketch an amplitude-modulated waveform.

3. Sketch a frequency-modulated waveform.

4. Describe the relationship of FM to PM.

5. Describe the formation of sidebands when a wave is modulated.

6. A carrier frequency of 200 KHz is modulated at a frequency of 20 KHz. What are the frequencies of the upper and lower sidebands?

7. Define "percent modulation of an AM wave" with the use of a diagram.

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